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ABSTRACT

The concept of the density of a material has an important role in elementary and secondary school science curricula, but it is a difficult concept to grasp. This project explores why this should be and whether there are some simpler, more accessible notions which can serve as the basis for building a concept of density in students' minds during the later elementary school years. The study explores the effectiveness of using computer models to help students build an understanding of density. This teaching strategy proved to be moderately successful with sixth graders. It was found that the majority of students did correctly assimilate this model in a way that supported their understanding of density as an intensive quantity and that they were able to articulate some relevant differences between weight and density. It was found that this distinction was necessary for children to have success at ordering by relative densities and in understanding a phenomenon such as sinking and floating. Appendices supply a description of the computer programs, worksheets, lesson plans, and interview instruments. (CW)

86-5

PROMOTING 6TH GRADERS' UNDERSTANDING OF DENSITY:

A COMPUTER MODELING APPROACH

Technical Report

July 1986

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Promoting 6th Graders' Understanding of Density: A Computer Modeling Approach

Technical Report
July 1986

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Weight, Density and Matter

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CHAPTER 1

INTRODUCTION

The concept of the density of a material has an important role in elementary and secondary school science curricula. Teachers have reported, however, that density is a difficult concept for even high school students to grasp. Our project explores why this should be and whether there are some simpler, more accessible notions which can serve as the basis for building a concept of density in students' minds during the later elementary school years. In particular, we are exploring the effectiveness of using computer models which visually depict size, weight, and density as distinct quantities, in helping students build an understanding of density. We have also extended the computer models to allow students to do simulations of sinking and floating experiments in a microworld in which the densities of materials are directly visible. Our long term goals are to investigate whether a modeling approach helps students build both a good qualitative and a good quantitative understanding of density as an intensive property of material kinds. We are also concerned with developing students' metaconceptual understandings of the role of models in science.

In our earlier work (Smith, Nov. 1984, June, 1985), we conducted two pilot studies designed to investigate the feasibility of using computer based models with elementary school children to build their understanding of density. We developed two different computer models for representing information about size, weight, and density. In the first model, weight was represented by the total number of dots in a rectangular shape, density was represented by the crowdedness of the dots, and size was represented by the total area of the shape (see Figure 1a, next page). In the second model, weight was represented by the total number of dots in a rectangular shape, density by the number of dots in a cluster, and size by the total number of clusters in a shape (note: the clusters were evenly spaced, see Figure 1b). We then investigated whether children could more readily think about the inter-relations among the three quantities depicted in the computer model than the quantities of size, weight, and density inferred from handling real materials. We found both these computer models encouraged children to think about the variables in question more quantitatively. However, children had difficulty correctly quantifying overall crowdedness, as presented in the first model, making it doubtful that it was a useful model for our purposes. The second computer model, however, was readily understood, and children showed a more sophisticated understanding of the inter-relationship among the quantities when dealing with this model rather than with real world objects. We concluded, then, that building a full-scale teaching intervention around the second type of computer model had genuine promise.

The work of the Weight/Density project this year has been in translating that promise into a reality. Several different types of work had to be done in order to develop a full-scale teaching intervention. First, the computer model itself had to be developed in order to make it usable for instruction. The model we had piloted had been static, with

Figure 1

Three Successive Computer Drawn Models Used to Depict Size, Weight, and Density in Our Studies

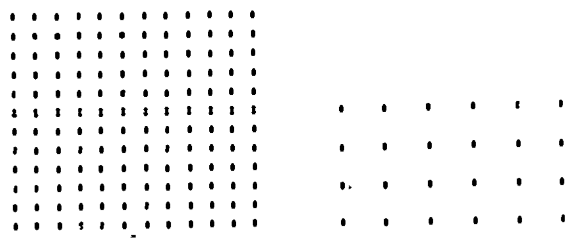

Figure 1a) First Pilot Study (Nov. 1984)

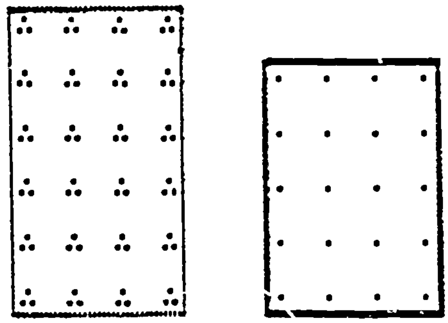

Figure 1b) Second Pilot Study (June 1985)

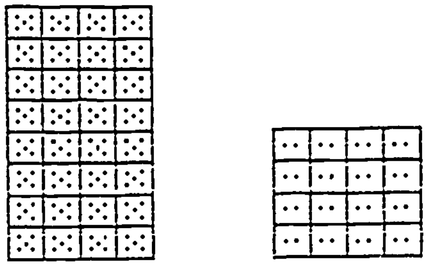

Figure 1c) Model Used in Current Pilot and Teaching Study (July 1986)

limited interactive capability. Work at the beginning of the year concentrated on making decisions about what the model should look like and developing a scheme for the interactive process. A first computer program was developed which allowed children to build objects of different sizes and materials, order the objects, change them and view or hide their structure.[1] The new model portrayed objects with a grid and dots representation (see Figure 1c). In this model, size was represented as number of squares in a grid, density as the number of dots per square, and weight as the total number of dots in a grid. Subsequently, two additional programs were developed which used the basic model to do simulations of sinking and floating experiments. Second, teaching activities and materials had to be developed to use in conjunction with the programs. In our teaching intervention, we wanted children to have experience working with real world objects as well as with the microworld, so that they could gain a deeper understanding of the phenomena in question and of the meaning of modeling. Third, we needed to pilot our teaching activities with a group of students, in order to find out how they related to our new computer programs and teaching activities and in order to select an age group for our teaching intervention. Finally, there was the teaching study itself, involving an entire 6th grade class in an elementary school in Watertown. Prior to this time, we worked one-to-one with students during our pilot studies. In our teaching, we made the transition to working with the class as a whole--an important step toward having an intervention which can actually be used by classroom teachers.

In Part I of our Technical Report for this year, we report on these separate activities, culminating in our discussion of the teaching study and its implications for further work. Chapter 2 describes the rationale and philosophy underlying the development of the computer programs and models. Chapter 3 discusses the pilot teaching study and how it affected our thinking in designing the teaching study. Chapter 4 is the main body of the report and describes the teaching study. This chapter is a draft of a manuscript we plan to submit for publication in the early fall. Consequently it contains a more extensive theoretical introduction to our work than we have provided in this first chapter. Finally, chapter 5 discusses how we plan to revise our teaching intervention in light of what we learned from the present study. The Appendices give more detailed descriptions of the teaching intervention itself and the stimuli used in the interviews.

Part II of the report is a working paper, prepared by Micheline Frenette. In it, she reports the results of an extensive pilot study she did this year designed to explore what features of the computer program may make it effective in helping children apply a concept of density to the phenomena of sinking and floating.

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[1] Daphna Kipman, Joseph Snir, and Judah Schwartz worked together in the initial stages of developing the software. Joseph Snir subsequently extended the software to do simulations of sinking and floating experiments.

CHAPTER 2

THE COMPUTER PROGRAM

The computer progrrams we have developed provide an environment where children can manipulate the different elements that play a role in the notion of density and its relation to the phenomena of sinking and floating.

Sinking and floating are rich, interesting and puzzling phenomena. Because they are governed by a limited number of independent variables, it is possible to build a compelling microworld in which children can investigate and learn not only about the specific phenomena, but about scientific inquiry and experimentation as well.

Because we wanted the program to react to user input from the keyboard in a way that would truly simulate the behavior of real objects and liquids, the relevant principles were embedded into the program. In other words, the computer model is scientifically accurate; the mathematical rules that the computer uses in calculating and portraying experimental results are the same rules that govern the phenomena of sinking and floating. Thus the learner can become familiar with the underlying principles and abstract mathematics of the phenomena through interaction with their dynamic numerical and visual representations on the screen.

In this chapter we describe the relevant concepts and variables, the graphic representations we chose for these concepts, the ways of interacting with these representations on the screen (the menu options), and the basic activities that the computer program can support.

The Physical Concepts and their Representation in the Computer Program

In devising a computer simulation, many decisions must be made about what is relevant to represent, how information should be represented and the kind of accuracy which is desirable. In what follows, we discuss the particular choices we made as well as our rationale for such choices. We believe some of these choices and assumptions are important to discuss with students as well if they are to understand how the model corresponds to the real world.

The simulation program we devised deals only with objects in the solid or liquid state, and it assumes constant temperature. (The temperature constraint will be lifted in the future.) Under these conditions, only three variables--size, weight, and density--are relevant to the phenomena of sinking and floating. Any two of these can be thought of as independent variables, which then determine the third. Usually, we take size and weight to be the independent variables. In the real world, we perceive and take measurements on objects, i.e., their weight and size, and then deduce or infer their density. These two extensive parameters of weight and size define, through a mathematical relation, the intensive quantity of density, which is the center and focus of our teaching effort.

When creating objects or building from materials, we use knowledge of the density of a material to figure out how much an object will weigh. In the computer program, the independent variables are density and size. These are the variables that can be selected and modified. The weight is determined by manipulating these two quantities.

Size

When we speak of the size of an object, the pertinent physical parameter is volume. As a three-dimensional quantity (length to the third power), volume must be represented in a symbolic way on the two-dimensional screen. We decided against designing the program to show perspective and three-dimensionality because we wanted the model to depict only the information that is directly relevant to the phenomena or topic at hand.

In the model, a unit of volume is represented by a two-dimensional square. Hence, there is a simple relation between the volume of an object and the abstracted represention of its size by the number of square units on the screen. This representation of volume can be used for an object of any shape, so long as one bears in mind that it is a symbolic and not a pictorial representation of size. We are concerned with volume and not with shape. All shapes are reduced to their rectangular (or cuboid) volume equivalents.

Since our main concern is to facilitate understanding of the principles and rules involved in sinking and floating and not to build a tool for exploring every real-life possibility, we have limited ourselves to a subclass of objects that are well-suited to our current purpose. For the time being, we have also limited the objects to homogeneous rectangles in their screen appearance; these rectangles stand for three dimensional objects, with the unseen dimension held constant.

The sinking and floating program shows two-dimensional representations of solid objects as well as of liquid in a container. The assumption is that both the object and the container of liquid, in three dimensions, would extend back away from the screen to the same extent. Of course, in real life, the object and container could not be exactly equal in this respect. We chose to ignore this discrepancy, however, because we wanted to keep any and all measurable size (volume) quantities visible and to avoid having hidden liquid or container volume behind the object. When the user gets numerical data, it corresponds directly to what he or she sees in the visual representation. Furthermore, this numerical data remains consonant with abstract principles as well as with actual (physical) measurement using suitable containers.

The idea of modeling and representations should be introduced to those involved in any attempt to use computer simulations as tools in science. The model's assumptions (what is relevant, how to best to represent, accuracy and compatibility with real phenomena) should be discussed and made explicit as well. Even if many of these ideas are not discussed with students, the model builder and the teacher should be aware of them. Clarity about the assumptions built into the model gives users the possibility of modifying or giving up some features as needed or desired.

In our pilot studies we discussed the meaning of size units a bit during class, modeling cubes made of 8 1cc blocks and discussing what was more relevant -- to portray shape or size (# of blocks). Although we designed the computer program to count blocks but not necessarily to show shape, we may change this or add more options to the program later. In any case, the program can be useful even before the concept of volume is discussed in class.

Weight

From the user's point of view, weight emerges from density, since the user first selects a kind of material and then determines an object's size. The weight of the object is represented visually (in a quantitative and consistent way) by the total number of dots displayed within the object's perimeter. This is not an atomistic picture of the solid. It allows the concepts of weight and density to be well-defined without any atomistic theory of matter. This symbolic representation could, however, be interpreted later in atomistic terms. As we now interpret it, each dot represents one unit of weight. (Later we might interpret the number of dots in a cluster as being proportional to the number of nucleons.) The total weight of an object is thus represented by the total number of dots that represent its weight in some arbitrary weight units.

Density

Density is represented as the number of dots in each size unit. This visual representation helps connect the notions of increasing crowdedness with increasing density. Since, at the moment, all objects created in the model are homogeneous, the number of dots per size unit is constant for any given object, thus conveying the notion of density as an intensive property of kinds of materials.

Material Kind

The computer allows users to define material kind in two independent ways: by density shown as dots per size unit or by color. So far we have described the dots per size unit option. When choosing the color option, the representation of weight and density by dots and dots per size unit are not visible. The object is presented as a solid color within its perimeter. Each material is a different color so that materials are distinguished by color, rather than dots per size unit.

In this mode there is no visually accessible representation of the variables of weight and density, but the specificity of materials is emphasized through another local property, color. The user can switch easily from one mode of representation to the other.

Other Features of the Program

Numeric Representations

In an effort to give multiple representations for the dimensions of weight, size, and density and to show the link between the visual displays and the values of the variables for each constructed object, the program allows the user to "collect" data about the size, weight, and density of any object displayed on the screen. When in the data mode, the data are displayed and updated as the user interacts with the program.

Process and Interaction

The program is divided into three parts. The user can move from one to the other through a common menu at any time.

Part 1: Modeling With Dots / Weight and Density

The first part is designed for manipulating the weight, density, size and shape (as long as shape is rectangular) of up to three different objects in three separate windows on the screen. This can be done in the dots or color mode, with or without displaying the numerical data (see Figure 2, next page). As seen in Figure 2, the student can "Build" an object (in one of three windows) and "Change" its material and size. One defines the object's mode of presentation (dots or color) through the "View/Hide" command. The "Collect data" command allows the user to display numeric information about the three variables independently. The user can also "Exchange" objects between windows.

This part of the program thus lets students explore the relationship among the three parameters and perform tasks that involve ordering, building, or modeling real life objects according to their different dimensions.

"Modeling with Dots" and "Weight and Density" are actually two versions of the same program. The only distinction between them is found when asking for data. The "Dots" program gives data with the labels "dots", "size units", and "dots per size unit", while the other version gives data in terms of "weight", "size units" and "weight per size unit." Thus the Modeling with Dots program affords some flexibility in designing activities which can deal with intensive quantities other than density (e.g., number of beads in a cup, number of pennies in a pile).

Part 2: Archimedes

In this part of the program the screen shows two distinct elements: an object of fixed size and a tub of liquid, also of fixed size. Students can perform "experiments" in which the object is immersed in the liquid (see Figure 3). This is a continuation of the first part of the program and enables the student to choose and manipulate several elements: (1) the object and the liquid in the container by changing the materials; (2) the modes of presentation; (3) data collection; and (4) when to perform "experiments".

The results of the experiments are shown visually on the screen. The object submerges to a depth that takes into account the relative densities of the material and the liquid. Liquid displacement follows accordingly. Numerical information about the level of submergence is also available.

The experiments can be done with both the object and the liquid represented in solid colors or in the dots mode. Once an object is immersed in liquid, however, it is represented as a solid color. This is to ensure a clear distinction between object and liquid borders. Even though the object is seen as a solid color, the "View" command enables the user to view the dot distribution in a small subsection of the object (see Figure 4).

Additionally, once an object is submerged, the rise in the level of the liquid is portrayed in a solid color. Since, in most cases, the increase in liquid level will not be an integer number of units, we felt it best not to complicate the screen display with partial or "open" squares (size units).

"Archimedes" is designed to enable students to explore the role of an object's and a liquid's densities in defining the outcome of a sinking and floating experiment. The approach we adopted was to keep the size parameter constant, thus concentrating student attention on the density parameter only.

Part 3: Sink the Raft

In this part, students can repeat the experiments in the previous section with one additional option. They can, in addition to all the other actions, change the size of the submerged objects and observe the effect such changes have on the outcome of the the sink-float experiments (see Figure 5). The data are continually updated, indicating the size, weight, density, and the portion of the object submerged as the user experiments. All objects and liquids in this section are portrayed in solid color.

In the future, we plan to gradually lift restrictions from the system and allow students to explore increasingly complex situations with regard to: shape, homogeneity of material, boat-like objects, waffle-like objects (with holes), and different size containers. We are also considering games which use submarines, mazes, canal locks, and balloons with weighted baskets.

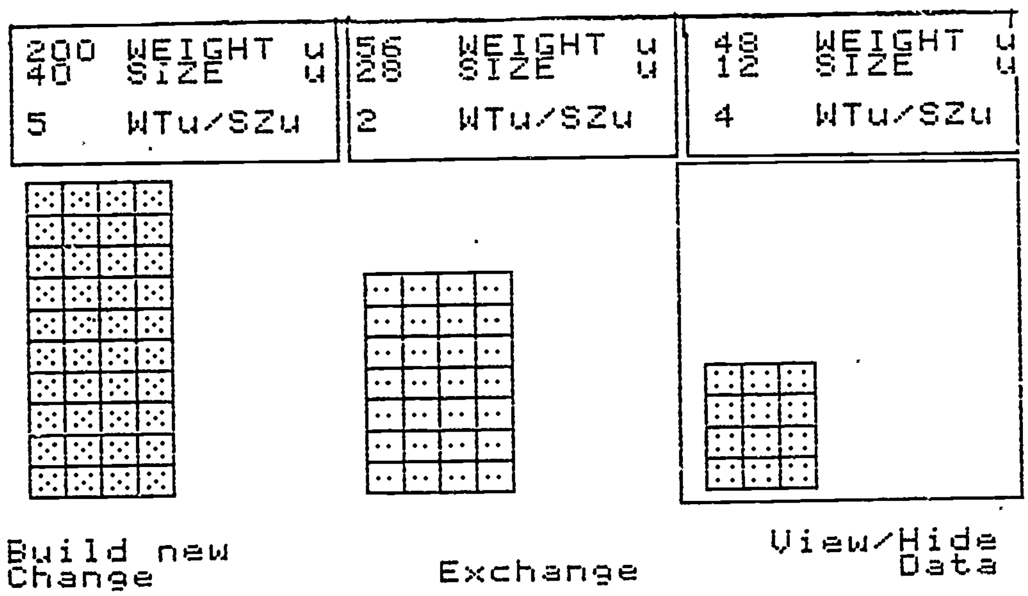

Figure 2
Screen Dump of the Modeling with Dots / Weight and Density Program

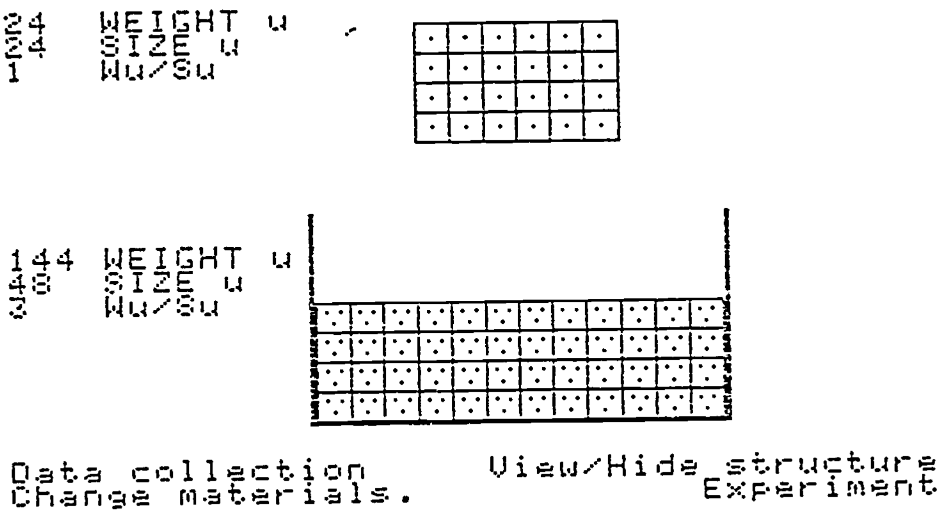

Figure 3
Screen Dump from the Archimedes Program

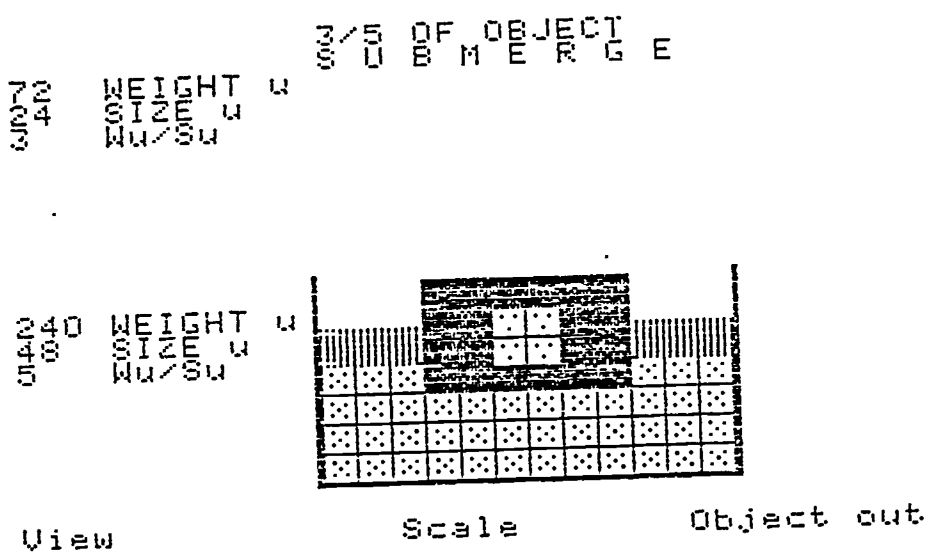

Figure 4. Screen Dump from the Archimedes Program

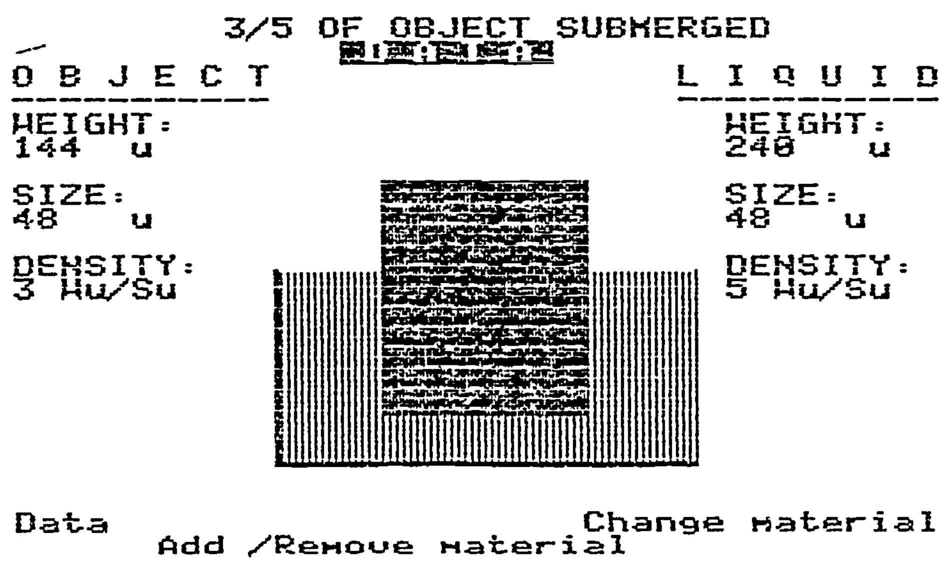

Figure 5. Screen Dump from the Sink the Raft Program

CHAPTER 3

THE PILOT STUDY

Our previous pilot work (Smith, June, 1985) had shown that elementary school children could work effectively with a computer microworld containing three quantities: the total number of dots in a rectangular shape, the number of dots per cluster in a shape, and the general size or total number of dot clusters in a shape, solving problems using information about two dimensions to predict a third. We also had found that 4th and 6th grade children could solve analogous problems with real world materials, using information about relative size differences of two objects and relative density differences to predict relative weight differences. Although children used more quantitative strategies in solving the computer problems than the real world problems, they showed a conceptual understanding of both types of problems. This result argued that children had at least some qualitative appreciation of relative density differences. Consequently, before embarking on our present teaching study we needed to pinpoint more exactly the limitations in young children's conception of density, and the age at which an intervention would be most timely. We also needed to explore how they reacted to our modified computer model, since in developing our computer model for use in a teaching study, we had made some changes in how the concepts of size and density were represented. Shapes were now composed of a uniform grid of equal size squares; in making a shape one could choose to work with building blocks which contained 1 to 5 dots per square. Size was thus represented by the total number of unit squares in a shape, while density was the number of dots per unit square. Weight was still represented as the total number of dots in a shape. These changes were made to introduce the notion of a unit size more clearly and to allow density to be represented as the number of weight units per size unit. Finally, we needed to see how children reacted to a richer array of problems with the computer microworld than we had originally piloted: ordering objects according to different dimensions, explicitly using the computer microworld as a model for real world phenomena, building objects in the microworld which satisfied certain constraints, using the microworld to derive certain mathematical rules and to experiment with sinking and floating, and so on. Thus, prior to undertaking a full-scale teaching intervention, we initially did some additional pilot work, working one-on-one with 12 children in the 4th through 6th grade.

This new pilot study was conducted at the Countryside School in Newton, Mass. We worked with 12 students: 4 each from the 4th, 5th, and 6th grades. The homeroom teachers for each grade selected four of their students for us. We asked them to select a diverse group of students (e.g., boys and girls, high and low ability students) and not simply their best students. The teachers complied with these suggestions, especially regarding ability levels, although they tended to select students whom they thought took some interest in science. The children were all from a middle to upper-middle class background.

Prior to instruction, each student was interviewed to explore his/her understanding of the weight, volume, and density of real world materials. The pre-interview also involved the child in constructing a model to explain the weight differences among objects. The teaching sessions involved the child in a tutorial with a teacher (a research assistant), in which the child used the computer program and explored real world materials. Each child received the same work and tasks, but the pacing was tailored to the individual needs of the child. There were two main activity sheets to be covered, which we initially thought would each take a 30 minute session. As it turned out, we usually needed more time to spend with each child (the sessions lasted 45 minutes and some children needed a third session). We also conducted one group session with all four students from each grade to discuss the nature of maps and the kinds of models they had constructed. Finally, half the students at each grade level received a post-test similar in content to the initial pre-interview, but with some modifications in how the tasks were presented. The other half went on to try the Archimedes program concerned with sinking and floating. In general, we selected those students who had felt most comfortable with the weight and density modeling program for trying out the Archimedes program.

The Interviews

The pre-instruction interview was designed to cover various aspects of children's knowledge about the concepts of weight, volume, density, and matter/material kind. Because the interview was exploratory in nature, we probed for children's understanding of particular words, their ability to order by the various dimensions, and their understanding of how to measure the various dimensions and to use relevant units for measurement. We wanted to identify both the strengths and limitations in their understanding of these various concepts, so we could better identify what should be the focus for the interviews and the teaching in the main teaching study.

We began by asking children about the weight of objects. Included in our tasks were: (1) judging whether a range of objects, including certain very light objects like a piece of cork and a grain of rice, had any weight and explaining how they knew whether something had weight; (2) ordering a group of objects by weight and explaining the basis for their ordering; and, (3) determining the weight of a particular object, to probe their ability to think about weight quantitatively and their ideas about units of weight.

We next asked children a series of questions about volume. We began by investigating whether they had heard of the word "volume" and if so what they thought it meant. We then gave them a brief definition of volume, and asked them to order a group of objects by their volume or "total size". This was followed by: (1) asking them to find the volume of a particular object, to probe their quantitative understanding of volume and their ideas about units of volume; (2) probing their understanding of water displacement as a measure of volume; and, (3) giving them some conservation of volume problems, using an abbreviated variant of Piaget's procedure of changing the shape of a piece of plasticene and asking them whether its volume had changed.

Questions about the density of materials followed the questions about weight and volume. We began by showing children two objects that were the same size but different weights (one made of aluminum and one made of steel) and asking them to explain how that could be. We asked them to "pretend we could look inside a small piece of each object, or take a powerful magnifying glass to see how these things are made inside" and then make a drawing of what they thought pieces of steel and aluminum might look like inside. After that, we asked children if they had heard of the word "density" and what they thought it meant. We gave them a brief definition of density, explaining that different kinds of materials have different densities, and if one material were denser than another, it meant that objects made of that material were heavier for their size than objects made of another material. This is clearly not the scientist's definition, but we thought it corresponded to the way they already thought about the density differences of materials. Thus it was a way of relating a new word to their existing conceptual structure. Students were then asked to order four same size objects made of different materials (styrofoam, wax, aluminum, and steel) according to their density, and then to add two new objects of different sizes but the same materials (steel and aluminum) to the order. This was one way of testing whether they thought that the density of a material did not vary as a function of the amount of material. Their understanding of the intensive nature of density was also probed in another way: students were shown a steel cylinder and were asked if it were cut in half, would the density of the half size piece be the same as the density of the full piece. Finally, children's ability to think about density differences quantitatively was probed. Children were shown an object made of steel that was one pound and a same size object made of aluminum that was one-third of a pound, and were asked how much denser steel was than aluminum. Two new objects made of steel and aluminum were then produced, and the question was repeated. Of special interest was whether students began by weighing the objects or whether they felt they immediately knew the answer from the previous demonstration.

The pre-interview concluded with some questions about matter and material kinds. Children were asked: whether objects were made of the same kinds of materials, whether if we cut up a piece of aluminum, it would still be aluminum, and whether we would ever get to a smallest piece of aluminum. Similar questions were posed about steel. For children who thought there was a smallest piece of steel and aluminum, we asked in what ways the smallest piece of steel would differ from the smallest piece of aluminum. These questions were designed to probe whether they could think about material kinds at a micro level. We also showed them two objects which were the same size, but made of different materials, and asked them whether the two objects had the same "amount of matter" in them. The same question was repeated for two objects of different sizes but equal in weight. These questions probed whether they had a clear way of conceptualizing the quantity "amount of matter" in their theory and/or recognized the amibiguity in this phrase. Given an atomistic conception of matter, one can distinguish the amount of mass in an object (which is proportional to the weight) from the number of particles of a given kind (which is more closely related to the overall volume of the object).

The post-interview was modified in several ways because, in the course of our working with children, we thought of new ways of probing children's understanding of density and modeling. These modifications, of course, make strict comparisons with the pre-interview impossible. However, our main purpose in the pilot study was to explore ways of asking children questions rather than strictly assessing the effectiveness of our teaching. One set of modifications involved the stimuli used for the ordering tasks. In the post-interview, we included objects of different sizes constructed out of 1 cc cubes of different materials. These new stimuli allowed us to see if children could use the strategy of comparing equal size pieces when making inferences about relative densities. This was a more difficult way of presenting the density ordering task than we had used in the pre-interview. In the pre-interview, children had first been given same size objects of different materials and then had to insert different size objects into their ordering. We wanted to test whether children now had a clear enough concept of density to succeed with this harder task. The other modifications were in how the modeling problems were posed. Children were given several modeling problems. First, they were asked to draw a picture which would "explain" how a 1 cc cube of copper, aluminum, and wood could all have different weights even though they were the same size. Then they were asked to draw a picture which would show how three different size objects all made of copper (a copper cube, a copper rod, and a copper penny) could have different weights but the same density. Finally, we explicitly asked them to use the notation of the computer model to represent similar situations.

The Teaching Sessions

The first teaching session focussed on their understanding of the three quantities in the computer program--total number of dots, total number of squares, and number of dots per square--and with developing their facility at controlling the commands of the program. We constructed some objects in the windows and asked children to order them in some way and to explain the basis for their ordering. We then pushed to see if they could find two other ways of ordering them. Finally, we introduced the three dimensions we had in mind and the data option of the program. Now children explicitly had to order objects according to the three dimensions, and construct objects which would meet certain specifications (e.g., make an object with the same number of size units as this, but with a different number of total dots, or make an object which has the same total number of dots as this object but which uses a different kind of building block). Finally, children were introduced to using the computer as a modeling tool. We first posed some intensive quantity problems for children involving real world materials (beads per cup or pennies per pile) and asked children to model these problems on the computer and explain the correspondence they used.

The next session was a group session to discuss the nature of maps and models. Children were shown four kinds of maps of Boston (a subway map, a street map, a road map, and a souvenir map) and were asked what each map represented, how it represented it, and whether one map was a "better" map of Boston. Then children were introduced to the idea of modeling in science where it was noted that models were in some ways like maps. A chart had been drawn depicting some of the models they had produced, as well as a model more consistent with the assumptions of the computer model (although it did not look exactly like it), and we discussed the strengths and limitations of the various models with children. Children were encouraged to think of ways of testing some of the assumptions of each model. For example, if a model explained the lightness of aluminum in terms of the object made of aluminum being hollow inside, children might suggest cutting the object in half in order to look. Or, if a model explained the heaviness of steel in terms of its dark color, one could investigate whether there were some dark materials (such as hard rubber) which were less dense than a lighter colored material like aluminum. The session generally concluded with student's seeing the limitations of some of the models they had proposed, although definitive tests of more atomistic models were not done. We did point out, however, that one strength of the atomistic models was that they allowed for quantitative predictions in ways that the other more qualitative models did not.

The third session was again an individual session in which students worked with the computer. The session began with a review of the three basic dimensions of the computer model. We then made a transition by focussing on the crowdedness of the constructed objects, seeing what parameters (if any) would affect crowdedness. (For example: Does crowdedness change when the size of the object changes?) Modeling of real world materials was finally introduced. Children were shown steel and aluminum cubes and asked to make an object out of steel cubes which balanced an object made out of aluminum cubes on the balance scale. After they saw that they needed three aluminum cubes to balance one steel cube, they were asked to represent these objects using the computer. With prompts they were led to make the two objects have the same number of dots (portraying their equivalent weights) while preserving the size unit relationships. Students were asked if it made sense to have dots stand for weight. They were then given the reverse task: selecting from a range of real world materials, the ones which had been represented on the screen. Finally, we asked some quantitative questions about the relative densities of materials, and pushed children to develop an explicit mathematical rule relating size, weight, and density.

From a pedagogical point of view, the teaching sessions involved presenting students with a series of structured problems, and helping them with a series of increasingly guided prompts to come up with the correct answer rather than directly telling them the answer. The whole situation was highly structured, involving problems that had been selected by us, rather than allowing students to explore the program in an open ended fashion.

Our approach in using the Archimedes program was considerably less structured and more open-ended. Here children were presented with the computer microworld and were essentially told to play with it to see if they could find a rule which would allow them to predict which objects would sink and which would float. There was some preliminary discussion of the meaning of the words "sink" and float". After children had formulated their rule, they were encouraged to test it out with real world objects. They were shown a small piece of wax which floated, and were asked to predict whether a large piece of wax would sink or float, based on their computer rule. Similarly, they were shown that a large piece of aluminum sank and then were asked to predict whether an aluminum paper clip would sink or float. After they made their predictions, they were allowed to test them. In some interviews, the session concluded with an informal discussion of the problem faced by Archimedes in determing whether the king's crown was genuine gold as well as some historical discussion of the man himself. Some students were given a modified version of this problem and were asked if they could tell the real material (clay) from an imposter (clay with cork hidden inside) by the way it behaved in water.

Observations and Conclusions

Based on our experience working with students in the pilot study, we discovered that students did not have even a good qualitative understanding of density as an intensive property of matter. Thus, we decided to focus our attention in our subsequent teaching study on building a good qualitative understanding of density, rather than focusing as heavily on understanding specific units of measurement and mathematical relationships. We also found that students could understand the computer as a modeling tool, and work their way through most of the activities we posed, although solving number based problems did not always generate good qualitative understanding of the phenomena. Fourth and fifth graders generally had less general knowlege of relevant phenomena (as will be elaborated shortly) and the math skills of fourth graders with multiplication and division could be pretty shaky, so we decided to begin our teaching study working with 6th graders. Finally, we were highly encouraged by children's enthusiastic response to the Archimedes program and their capacity to formulate a rule for sinking and floating in terms of density, and hence decided to make it a regular part of our teaching unit.

Looking more specifically at children's responses in the pre-interview, we found evidence of age differences in the following beliefs: weight is a property of matter, the volume of an object is not affected by a shape change, the smallest piece of steel and aluminum differ in weight, and matter is particulate and the spacing between particles can explain density differences of materials. In general, there was a steady increase with age in the proportion of children holding each of these beliefs. In our small sample, only 25% of the 4th graders thought that the styrofoam, cork and grain of rice all weighed something (and none of them justified their answers by asserting it had to weigh something because it was made of matter), while all of the 6th graders thought these objects had weight and explained their judgment in terms of their belief that all matter had weight. Similarly, only 25% of the 4th graders clearly understood that changing the shape of plasticene did not affect its volume, while all of the 6th graders did. 25% of the 4th graders believed that the smallest piece of steel and aluminum would differ in weight, while all of the 6th graders did. Finally, none of the 4th graders used atomistic explanations of the weight differences of objects, while half of the 6th graders did. In regard to this last point, it should be noted that the 6th graders had received some explicit instruction in the atomic theory of matter already in school, which may account for this age shift. The responses of children at the earlier ages were quite various and inventive, including: the aluminum object is hollow inside while the steel object is full or has a weight in it, differences in the surface markings of the two objects accounts for the weight difference, differences in the hardness/softness or dullness/shininess of objects accounts for their weight differences, the steel has magnets inside which pulls it to earth while the aluminum is more free floating, the steel has sinking cells inside which sink to the bottom and press down on the hand while the aluminum has floating cells which rise to the top and push up, and so on. One striking feature of the models produced by most of the fourth and fifth graders was that they did not presuppose that materials were homogenous and uniformly distributed throughout the object.

In one respect, however, children at all ages were the same: very few realized that two different size pieces of steel (or aluminum) have the same density. We probed children's understanding of density as an intensive quantity in two ways: by asking them to insert two new pieces of steel and aluminum into their ordering of the density of the materials (they had already ordered same size pieces of wax, steel, and aluminum) and by asking them to compare the density of a large piece of aluminum with a piece created by cutting it in half. Only 25% of the 4th and 5th graders combined responded by consistently asserting that the density of objects made of the same materials was the same, while none of the 6th graders had this basic insight. Thus, coming to understand the intensive nature of density as a quantity was not an aspect of children's understanding that was spontaneously improving during these years (or improving as a result of whatever science instruction children had).

Finally, there was another respect in which there were no uniform age trends: in children's developing a clear way of conceptualizing the quantity "amount of matter." No child responded by saying that it was an ambiguous question, and that there were two distinct ways of construing "amount of matter." Half the 4th and 6th graders simply used the sizes of the objects to infer the amount of matter in them while half the 5th graders consistently used the weights of objects to make this inference and two children (one 5th and one 6th grader) always choose the steel object as having more matter. The rest of the children picked the heavier object as having more matter when they were the same size and the larger object as having more matter when they were the same weight (or were unsure what to do in the latter case).

During the teaching sessions, we learned that children could order objects in the computer microworld by the three dimensions. They could also solve simple problems requiring them to think about the interrelations among the three quantities, although some of the 4th graders were very shaky in working with multiplication, and thought the problems through by counting or using addition and subtraction. All the children had a pretty clear understanding of maps and agreed that no map was better than the others, but just served different uses. All the children were also comfortable with using the computer as a modeling tool for the beads and pennies problems, and able to use it in a qualitative fashion to represent the density differences between steel and aluminum. However, they did not think about the density differences in a precise quantitative terms without further prompting--i.e., they simply wanted to show that steel had more dots per size unit than aluminum and did not worry that it has three times more. Some of the better and older students were able to understand density more quantitatively and to use the computer model to work their way up to a mathematical formulation of density in terms of weight per unit volume. Here it genuinely seemed that the computer representation helped them. When we asked them to formulate a general rule relating the different quantities, they at first looked lost, but then were able to understand the question working with the computer representation and transfer their solution to the real world objects.

Half of the students at each grade received the post-interview. All the children's spontaneous models had changed from the pre-interview in ways that showed a better grasp of density, but no child directly made use of the computer model in the form we had presented it. The two fourth graders moved from depicting dense objects as full or having a weight in them and less dense objects as hollow, to showing objects as uniformly filled with matter of varying shades of gray. Objects made of the densest material were depicted as darkest, while objects made of the least dense material were depicted as lightest. Further, objects of different sizes but the same density were depicted as having the same intensity of shading. While these drawings made no attempt to deal with the dimensions of size, weight, or density quantitatively (by depicting explicit size and weight units), they had shown a grasp of the fact that pure materials are uniformly distributed and they had, at least implicitly, depicted density as an intensive property of materials. The older children all made more attempt to consider explicit units, but not always completely successfully. For example, one 5th grader when given the problem of modeling how the different size pieces of copper could have the same density explicit'y argued that if you took an equal size piece from each they would weigh the same (note: he actually developed this insight in the course of working through this problem in the post-interview; he was initially somewhat perplexed by it and had not been able to model the three objects of the same size but with different densities). And both the sixth graders, in modeling the three same size pieces of different densities, explicitly noted that denser materials had more stuff (particulately represented) packed into the same size piece. One also went on to successfully model the sitution of same density but different sizes; the other became confused. When we asked children to use the computer notation to depict objects of varying sizes (and weights) but made of the same material, now all of the older children and some of the younger children as well correctly portrayed the objects as made of the same building blocks (and hence having the same density), but differing in numbers of size units. Thus, most children had assimilated how to use the model correctly.

Turning to the ordering tasks, children were in general less successful. Here only one of the students (a 6th grader) was able to order the objects correctly by the density of materials and one 4th grader succeeded with some prompting (none had done it correctly in the pre-interview). Of course, the ordering tasks came first in the post-interview, and children might have done better if we had asked them this question after they had worked on the modeling problems. Nonethless, it revealed how shaky students were in understanding density as an intensive quantity. We suspected that in our concern for having students deal with quantitative aspects of the inter-relation of the three quantities, we had not stressed building a qualitative sense of density as an intensive quantity-- that is, a quantity which is defined locally and is not affected by the total amount of matter in an object. We concluded that in our subsequent teaching efforts we should put greater stress on qualitative understanding of the model and explicitly teach procedures for ordering by relative density (which can be understood in terms of the model). In addition, we decided to change the way we asked students to construct their own models. Rather than ask them to invent an explanation of why same size objects have different weights (with an emphasis on thinking about materials at a microscopic level), we decided to elicit their ideas about some of the factors that affect weight and then ask them to draw a picture which represents those ideas. We made this change primarily because we had decided not to interpret our model in atomistic terms at present. Since most children did not spontaneously believe in atomistic conceptions, and since it would be too complicated to present a range of experimental support for such a wide-ranging theoretical assumption, we thought it might promote greater assimilation of our model to present it on a level more compatible with their conceptual framework. In our fuller teaching unit we planned to introduce students to a range of models to emphasize the dynamic and changing quality of models rather than their being construed as truth.

The results with the children who were given the Archimedes program were more encouraging. With this subgroup, all but one (a 4th grader) were able to formulate a general rule in terms of density and understood what the rule would imply for real world objects. Also, the children greatly enjoyed this session and worked easily with the program. Since understanding the program depended upon their understanding our underlying model, this result provided some evidence that this group of children had understood some of the previous lessons. They also were some of the students who had shown greater facility with the model during the teaching sessions.

Overall, we were able to pinpoint from the pilot study those qualitative aspects of understanding density on which to focus in the teaching study. We decided to work with 6th grade children because although they did not yet have these qualitative understandings, they did have much knowledge relevant to understanding our model. We also decided to make the teaching excercises less quantitative in nature since facility with number problems did not always yield qualitative insights, and to include the Archimedes program in the basic unit since it was so motivating. Finally, we decided that it would be more useful to begin with a non-atomistic interpretation of our model, since many students do not yet have atomistic conceptions, and that it was important to stress evaluation of models as useful or not useful for some specific purpose rather than to present them as "truth".

CHAPTER 4

THE TEACHING STUDY

Introduction

The purpose of the present study was to determine whether we could bring 6th grade students to understand more clearly the distinction between weight and density and to apply a concept of density to situations of sinking and floating. In keeping with the recent literature on students' alternative conceptual frameworks in science (Driver & Easley, 1978; Driver & Erickson, 1983; Novak, 1978, 1982; Champagne & Klopfer, 1984), we assume that children come to science class with existing conceptual frameworks which need to be engaged and modified in the course of science instruction. The challenge is to understand what (if anything) students already know about density, how this knowledge is organized, and how to use this information about student starting points to create a successful teaching intervention.

At first glance, previous research supports differing conclusions about the age when children are "ready" for instruction in density. Piaget and Inhelder considered the construction of a concept of density and the formulation of the law of floating bodies to require the development of formal operational thought in adolescence. In their pioneering work, The Child's Construction of Quantities (1974), they trace the development of the child's concepts of size[1], weight, and density and relate the concepts both to the development of children's atomistic theories of matter and to the development of logical thought. They argue that initially, in the preschool years, the pre-operational child has an undifferentiated concept of weight, size, and amount of stuff. At this point, the child cannot quantify this intuitive concept and hence is unable to realize that the size, weight, and amount of stuff in an object remains the same when simply the shape of the object is altered. With the development of the quantifying operations of concrete operations in the early elementary school years, the child first differentiates a notion of amount of stuff from weight and size. He now assumes, for example, that the amount of clay in a ball remains the same when the ball is rolled into a sausage shape, although he still believes that these transformation change the size and weight. Subsequently, he comes to quantify (and conserve) weight as well as amount of stuff. At this point, he makes a clear differentiation between weight and size. However, the child does not yet clearly distinguish between weight and density, and does not assume the underlying stuff to be atomistic in form. Finally, with the onset of formal operations, the child constructs a formal concept of volume (which he can now conserve), relates weight to volume in constructing a concept of density (using proportional

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1 We use size here and throughout to indicate the volume of an object (children might be aware of the size of an object before knowing the definition of its volume) and density for its specific weight (i.e., the weight of a unit volume).

reasoning schemes), and formulates an atomistic conception of matter in which density is understood in terms of schemas of compression and decompression.

In related work about the development of the child's understanding of sinking and floating, Inhelder and Piaget (1958) also show that it is only at adolescence when children can formulate the law of floating bodies by stating that objects float if their density is less than that of water, and sink if their density is greater than water. They argue that the formulation of this law requires formal operational thinking because it involves density (a formal operational concept) and because it involves imagining a hypothetical entity (the amount of water equal in volume to the volume of the object). Earlier, children invoke multiple explanations for why things sink and float (because of its weight, size, shape, etc.) and are unable to come up with a single formulation. Gradually, they come to have some intuitive notion that different materials have different specific weights, but are unable to use this notion to come up with a coherent and unified explanation.

More recent work within the Piagetian tradition has put greater emphasis on concrete operational "precursors" of the density concept than Piaget and Inhelder did. For example, Emerick (1982) writes: "Data from the present research indicate that density is a concept that is constructed by a child over a period of years, probably beginning as early as when he or she is able to squeeze objects and to recognize that objects are made of different substances."(p. 177) These data included the fact that some subjects had the intuition that what an object is made of affected whether it would sink or float, and that if an object sinks or floats, then an object made of the same material will react that same way regardless of the size or weight of the object. Further, the child has some intuitive notion of specific weight. In fact, Emerick's data are not that different from Piaget's original data; Piaget too noted that in the late concrete operational stage children had these intuitions. But he explicitly claimed that the child still did not differentiate weight and density; thus, Piaget felt that formal operations were essential in making this differentiation. Bovet et al (1982) made modifications to the traditional volume and density conservation tasks and argued that concrete operational children can in fact conserve volume and can conserve an intuitive density concept at around the same age that they conserve weight (i.e., ages 8 -10). These results are more novel, and were not clearly anticipated in Piaget's earlier work. Nonetheless, what Bovet et al call an intuitive density concept is the child's realizing that different substances have different specific weights and that the differences are preserved with successive halvings. They provide no evidence that the child differentiates this notion from absolute weight. Thus, it is still unclear from this more recent work whether the child's intuitive density concept is part of his weight concept or distinct from it.

Smith, Carey, and Wiser (1985) were specifically concerned with testing Piaget's claims that concepts of size and weight and weight and density undergo differentiation during middle childhood and early adolescence. Like Piaget, they felt that conceptual differentiation was an important kind of change that occurred in cognitive development and needed to be studied in the overall context of theory change. Unlike Piaget, however, they did not attempt to study differentiation within the framework of his logical stage theory, and they used a different range of tasks to study children's ability to use these concepts. They also placed greater emphasis on earlier developments within children's matter theories than atomism--children's formulation of a clear notion of material kind.

In their work, they found that even preschoolers clearly distinguished between size and weight as dimensions; further, although there was evidence that children had an undifferentiated weight/density concept in the late preschool and early elementary school years, they found that by ages 8-10, children do develop a precursor of a more formal density concept which is distinct from their weight concept. At this age, most of the children in their sample had two distinct senses of weight available to them--heavy and heavy for size--and use heavy for size in generalizations about materials and heavy when considering the weight of the total object. They realized that a large aluminum object can equal a smaller steel object in weight, while at the same time noting that the aluminum is a lighter material than the steel. They realized that an object made of a heavier kind of material can be lighter than an object made of a lighter kind of material. And they correctly sorted objects into steel and aluminum families by making weight judgments relativized to size (the objects were covered with contact paper so that visual cues could not be used). Further, their understanding of material kinds had advanced to the point that they now thought of objects as constituted of materials at every point (and not just as constructed from materials) and they were beginning to distinguish between some of the properties of materials which only emerge when they are in bulk quantities (e.g., some of the surface markings and characteristics) and properties of materials which hold at a micro level (e.g., having weight and size). From their data, Smith et al argue that children are beginning to develop a sophisticated matter theory during the middle elementary years--albeit not yet an atomistic theory--which calls for children to distinguish between two senses of weight. Thus, the differentiation between weight and density begins well before adolescence and does not require an understanding of atomism.

Of course it should be noted that the density concept possessed by elementary school children is still quite limited and different from that of scientists. Nonetheless, it is significant that such a precursor concept seems to develop naturally, without formal instruction, since the topic of density is not broached in the curriculum until grades 5 and 6 at the earliest (and frequently not even until grades 7 ,8, or 9). Piaget and Inhelder were concerned with the child's ability to formulate a density concept, mathematically, more like the scientists. The fact that children typically achieve such an understanding in adolescence, at a point where they have been taught such a concept in schools, raises a number of important questions. To what extent is instruction necessary for children to progress beyond their intuitive density concept? What are some of the steps they take in assimilating the scientist's conception of density? How can instruction best enhance such further development? How far can elementary children progress in constructing a formal concept of density and in formulating a law to explaining sinking and floating?

We would argue that without some explicit instruction it would be difficult for most children and adults to go beyond their intuitive density concept and to understand sinking and floating in terms of relative density. A number of researchers have found that adolescents with little prior science instruction and adults whose science instruction came a while ago did not initially formulate the law of sinking and floating in terms of a concept of density (Rowell & Dawson, 1977a and 1977b, 1983; Duckworth, 1985); Cole & Raven, 1969). For example, Rowell & Dawson (1977a and b) report that only one ninth grade student explained sinking and floating in terms of density prior to instruction. And Eleanor Duckworth (1985) reported that it took over 8 weeks of extensive experience experimenting with sinking and floating for a group of adults to formulate an understanding of sinking and floating using a concept of density. Further, Rowell and Dawson found that even with explicit instruction 9th grade students had difficulty learning to accept that the densities of pure materials defines a constant for those materials (1977a) and many students failed to understand sinking and floating in terms of density (1983).

These results are particularly interesting for two reasons. First, they suggest one way in which student's intuitive density concept may be deficient: density may not yet be clearly conceptualized as an intensive property of materials--one which does not vary as a function of the amount of material in the object. In our pilot work with 6th grade students, we also found evidence that students did not yet have this understanding about density. Second, the results suggest that students may have important conceptual resistances to learning the contemporary scientific concept of density and applying this understanding to the phenomena of sinking and floating.

A consideration of some of the difficulties scientists had historically in understanding sinking and floating highlights the complexity of the problem of asking students "why certain things sink and other things float". It also points to the importance of distinguishing among several elements and stages that compose such an understanding. One element is the understanding of the concept of density per se as an intensive specific property of matter. This can be done, as was done in ancient times, by merely recognizing the existence of such a property of matter, without relying on an atomistic theory as an explanation for the density differences of different materials. Students, unlike the ancients, seem to have difficulty even at this level. A completely separate issue is whether one can use this recognized property as an indicator for predicting if a certain object will float or sink. Here it is important to distinguish formulating a predictive rule which uses density as an indicator and thus enables them to know when an object will float in a given liquid from explaining why and constructing a theory to explain the phenomena. Such a theory will have to rely on concepts and laws of hydrostatics or on energy considerations. It is interesting to note that Archimedes in his work tried to formulate an understanding of sinking and floating without using a concept of density explicitly (instead he thought in terms of balances and the relation between the weight of the whole object and the weight of the amount of water displaced by that object). Further, his famous rule only covers limited aspects of the phenomena of sinking and floating (see The Works of Archimedes, translated by T. L.

Heath, Cambridge University Press, England, 1897). Galileo attended to this problem in his work ("On bodies that stay atop water or move in it," 1612, in Cause, Experiment and Science translated by Stillman Drake, University of Chicago Press), enlarging the law to a more general case, but again was not able to give a complete explanation (see Snir, in preparation, for further details about the historical development of the concept of density and the law of sinking and floating). This historical perspective reveals that these scientists did not look for predictive rules relating sinking and floating to density in their work (or even mention such rules). Further, it reveals how hard it was for some of the best scientists of the day to give a complete explanation of these phenomena, and how such explanations require use of many other physical laws and concepts. Certainly, then, we cannot expect students to construct such complete explanations on their own.

How, then, should instruction about density proceed? Previous work has found that highly formal approaches are not well understood by many average 9th grade students (Rowell & Dawson, 1977a). For example, Rowell and Dawson had students weigh and measure the volume of many different pieces made of the same material and graph the results. From this experience, many children had difficulty formulating the generalization from this experience that the densities of specific materials were constant (under standard conditions). Further, Cole & Raven (1969) found that, among their older group of students (8th graders and adults), direct instruction in the correct principle for understanding sinking and floating was not nearly as effective as instruction which engaged and challenged students' prior beliefs about sinking and floating (i.e., involved students in excluding irrelevant principles which they previously had thought were relevant). Significantly, there was no evidence that the younger children in their sample (7th graders) benefited from any form of instruction about sinking and floating, but this may have been because they made no attempt to teach children an explicit concept of density, in a way that built on their natural concept of density, prior to having them explore the phenomena of sinking and floating. The knowledge level of the 7th graders about related phenomena was considerably less than the knowledge level of the older students.

We believe that younger children are ready to understand density as an intensive quantity distinct from weight and apply this understanding to sinking and floating, if they are taught about density in a way that builds on their natural concept and their understandings of material kinds. Their natural concept of density is articulated in terms of heavy for size--an imprecise notion (not yet weight per unit volume)--which does not lend itself to ready quantification. Because students have no clear notion of a unit size, heavy for size cannot define a quantity which is distinct from weight--it remains a more qualitative notion. Thus, children do not have two distinct quantities--and in tasks which call for them to think about density in a more quantitative fashion, they may revert to using weight. Second, although children have an intuitive way of conceptualizing the density of materials, they do not have an atomistic conception of matter and may not yet be able to generate models of materials which allow them to separately portray size, weight, and density as quantities. For example, many children conceptualize denser materials as "thicker" rather than as "more crowded with particles" or as having "heavier particles, uniformly packed". Their visual model in terms of thickness may serve to confound the distinction between size, weight, and density, rather than sharpen it, since thicker materials are often thought to be wider than less thick materials. Providing children with an alternative visual model of density (which portrays density, size, and weight as distinct quantities but which is not yet presented in terms of atomistic conceptions) may help them to see density as a distinct quantity from size and weight.

In our work we try to develop 6th grade children's understanding of density as an intensive quantity through teaching activities which involve them in constructing their own models and working with a presented model. Visual models are concrete and can depict the interrelations among size, weight, and density directly rather than solely in an algebraic or numerical way. They also allow us to present ideas about standard units which are conceptual in nature and do not presuppose a full understanding of volume. Thus, models can be used to build a qualitative as well as quantitative understanding of some of the important properties of density. In addition, modeling is an important activity for scientists, but one which has been little used in science teaching. Modeling is of central concern to any attempt to use computer simulation in science education, since such simulations are, after all, models. Students may misinterpret the complex relation between the computer model and the real world unless they have some awareness of the process of modeling. Therefore, quite apart from the need to teach about density, the curriculum needs to develop in children an explicit understanding of the nature of models and how they function as a tool in science. Working with models of density is a good place to start such instruction, precisely because it is such a limited and simple physical situation.

In our present teaching, we begin with a simple computer model representing only three quantities: the size of objects, the weight of objects, and the density of the materials the objects are made of. Children build objects on the computer screen where variation in the size of the objects is represented by the number of standard sized building blocks that are used in its construction, variation in the weight of the objects is represented by the total number of dots in all the building blocks of which the object is composed, and variation in the density of the material is represented by the number of dots per building block (there are five types of building blocks, ranging from 1 dot per block to 5 dots per block). The computer program is also restricted to constructing only objects of uniform density and rectilinear shape to avoid the problems of introducing objects of mixed density at this time (e.g., objects with holes in the middle, objects made of different materials). The model represents objects as continuous (all the building blocks are flush with one another) and no attempt is made to interpret the model in atomistic terms (dots simply portray the amount of weight packed into a certain size unit; they are not described as nucleons). At this point, the idea is simply to show students (visually) that some materials have 2, 3, 4, or 5 times the weight per unit volume as other materials, since a deeper explanation is not needed for an accurate description of the concept of density. The program also directly displays <u>data</u> about the size, weight, and density of objects that have been constructed and permits students to conduct simulations of simple experiments in which they can visually perceive the quantities of density, weight, and size, as well.

In summary, the twin purposes of the present study are: (1) to see if we can help 6th grade students build a good qualitative understanding of density as an intensive quantity using a modeling approach and teach them to apply this concept to the phenomena of sinking and floating; and (2) to investigate how 6th graders spontaneously model density and how ready they are for metaconceptual instruction about the nature of modeling.

Methods

Subjects

This teaching study was done with a sixth grade class at the West-Marshall School in Watertown, MA. There were 19 students in the class: 7 girls and 12 boys ranging in age from 11 to 13 years. One girl was absent from school during the week the pre-interviews were done. Therefore, although she participated in the teaching experiences and the post-interview, her data could not be included in the main analyses.

Watertown is a suburb of Boston and the students of the West-Marshall school are mostly from families of low to middle income. The school is both an elementary and junior high school and its population is ethnically diverse (e.g. Greek, Armenian, Irish-American, Italian-American, Scottish-Canadian, French-Canadian).

Procedures

Overview

We worked directly with students in three stages: first conducting individual interviews; then presenting instructional material to the entire group in a series of eight lessons; and finally conducting individual interviews once again. We present here a brief overview of this work; in sections that follow, we will describe each stage in greater detail.

Each student was interviewed privately before the teaching sessions began. The interviews lasted 45 minutes to one hour. There were usually 2 adults present - one interviewer and one recorder. Occasionally an observer was present and on two occasions the interviewer also acted as recorder. (These interviews will hereafter be refered to as "pre-interviews")

Questions were designed to gather information about the students' ability to: distinguish among the dimensions of size, weight, and density; order objects according to these dimensions; describe and explain similarities and differences among objects relative to these dimensions and to represent these aspects graphically; and give explanations for the sinking or floating behavior of various solid objects in liquid.

The teaching sessions involved the class as a group. There were eight instructional periods and each lasted from 1 to 1 and 1/2 hours. They were held twice a week on the average, and were presented by the research team.

The students' regular teacher was usually present during class, observing, overseeing the smooth running of class, and assisting with the handling of student questions.

Central to the teaching sessions was the use of our computer programs which offer a visually accessible and mathematically accurate model of size, weight and density, and a microworld in which to investigate sinking and floating phenomena. Students also handled real materials, witnessed demonstrations, participated in discussions, and filled out worksheets which at times called for them to copy their computer generated images onto paper.

Classes took place in the school's computer lab. Students either sat in a semi-circle with their backs to the computers - facing the teacher, blackboard and demonstration desk - or worked at the computers in pairs, each pair having its own Apple IIe. Occasionally, some pairs preferred and were able to break up and work alone. When activity worksheets were given, pairs worked through the exercises together, with each student filling out his or her sheet separately.

There was one exception to this general format. One lesson was given to only two students at a time, using a computer set up in the library. The purpose of this was to give more individualized attention to students about mid-way through the intervention.

Students were again interviewed individually after the intervention ("post-interviews"). The questions were the same as those on the pre-interview with just a few exceptions. The time between pre- and post-interviews was approximately 5 weeks.

The Pre-interview

The pre-interview was divided into 3 parts: ordering objects along the dimensions of weight, size, and density; exploring ideas about what makes various objects weigh what they do and representing these ideas graphically; and articulating some rules, predictions, and explanations concerning the sinking or floating behavior of objects.

The Ordering Tasks. In this part of the interview, students were first given a very small rubber cube and asked if it had any weight at all. This was intended to elicit whether or not students believed that matter must have some weight or mass, even if the object's felt and/or scaled weight was insignificant. If a student did not think the object had any weight at all, he or she was given 10 such cubes and asked whether these had any weight.

We then proceeded to ask for three separate orderings of various objects: by weight; by size; and by "the heaviness of the kind of material objects were made of, that is by the density of the material." Objects were selected so that the three correct orderings would be quite different.

Based on the pilot data, we did not assume that students knew the meaning of the word "volume", so we described volume in terms of "total size", "size all around", and "the amount of space it takes up."

Students were asked if they had heard of density as a separate question. Since most students had not heard of density, we offered the following clarification: "Some objects are made of a heavier kind of material than others. I would like you to place these objects according to the heaviness of the kind of material they are made of, that is according to the density of the material."

Prior to the density ordering task, students were asked to group objects according to the materials of which they were made. This was one place in the interview where correct answers were supplied if students made mistakes. We also went over the actual names of each kind of material. Since density is constant for a given material, we felt that clarifying material kind groups would help us avoid some confusion when diagnosing responses. (For example, if students did not put objects made of the same material together as having the same density, it could not be because they were not aware that they were made of the same material).

The stimuli for the ordering tasks were grouped into two sets. The first set consisted of small equal size (1 cc.) cubes made of rubber, steel, aluminum, and copper. Objects were constructed out of these cubes by placing them side by side or on top of one another. The objects were: a single aluminum cube; a group of 5 aluminum cubes laid flat in a line; 5 aluminum cubes arranged as a modified rectangle standing vertically; 3 steel cubes laid flat in a line; and 7 rubber cubes laid flat in the shape of a modified rectangle. Students were told to consider the arranged cubes as distinct objects, but that they could take them apart if that would help them complete the task. We also provided a balance scale, a postage scale with a 5 lb. capacity and a tape measure for their use.

The second set of stimuli consisted of 3 cylinders, 1 1/2 inches in diameter. Two were of aluminum (3 and 6 inches tall) and one was of steel (2 inches tall).

Students were asked to produce orderings of the first set (cubes) and then to add the second set (cylinders) to the order. In this way, the number of objects students would have to order at one time was reduced. Furthermore, the cube-type objects afforded ordering strategies that could not be used for the solid cylinders. (For example, equal size cube samples could be taken from each object and weighed in order to determine their material kinds' relative densities.

<u>Questions About Weight and Modeling Tasks</u>. Students were given a set of 5 cylinders - 1 made of wax, 2 made of aluminum, and 2 made of steel. Included in this set were: objects of equal size but different weights and materials (1 aluminum, 1 steel, 1 wax); objects of the same material, but different sizes and weights (2 steel, 2 aluminum); objects of equal weight, but different materials and sizes (1 aluminum, 1 steel).

Students' ideas about factors which might affect an object's weight were first elicited verbally. They were then asked to represent these ideas on paper, using their own "picture code". We specifically tried to concentrate attention on material and size as relevant factors by asking such questions as: "These two steel objects weigh different amounts. How could that be?"; "These objects are the same size, but weigh different amounts. How could that be?"; "These two are different sizes, yet they weigh the same amount. How could that be?"

After they handled the objects and answered the questions, the students were given a piece of paper and 8 colored pencils and asked to produce a drawing. They were told that their drawings did not have to look exactly like the objects, but rather they should just try to represent the information as they saw fit, using their own code, focussing on the ideas we had just talked about. Students were reminded that we had talked about similarities and differences with regard to the objects' size, weight, and material.

When finished, we asked them what information they had represented, how they had represented that information, and if they thought their code was useful.

<u>Questions About Sinking and Floating</u>. The third part of the interview entailed making predictions about whether objects would sink or float and explaining how the same object (a piece of lucite) could sink in one liquid (water) and float in another (salt water).

Students were first given a set of eight objects made of four different materials. There were two sinking materials (plasticine and lignam vitae wood) and two floating materials (hardened glue and pine wood). One large and one small object composed of each material were given. Included in this range of objects were two pieces of wood with equal size dimensions (one sinking and one floating), relatively heavy floating objects, and relatively light sinking objects.

Students were given a tub of water and asked to investigate how they would behave in water. They were asked to comment on what kinds of things sink and what kinds float, and then to come up with a general "rule" which could be used to predict whether something would sink or float.

We then asked students to predict whether a particular object would sink or float based on their experience with a different size object made of the same material.

Finally, students were presented with a small piece of lucite and two plastic cups filled with equal amounts of liquid. One cup had red liquid (colored water) and the other, blue liquid (colored salt water). They were to place the lucite first in one liquid, then in the other, and offer an explanation as to why it floated in one liquid and sank in the other.

The Teaching Intervention

The Software. During the course of the teaching, three computer programs that we designed were used: Modeling with Dots/Weight and Density, Archimedes, and Sink the Raft (see chapter 2 for a description of these programs).

Real World Materials. A number of real objects and materials were used during the teaching sessions.

A range of steel and aluminum pieces were used that included: equal size cylinders of each weighing 1 lb. and 1/3 lbs.; several 1 cc. cubes of each; a very large aluminum cylinder, weighing approx. 5 lbs.; smaller cylinders (approx. 1/4" diam. by 2" tall) which were equal in size with more cylinders made of wood, hard rubber (vulcanite), and brass. Brass cubes, cork, other wood pieces, and clay were also used.

Students had rulers and pencils. We provided two scales (balance and postage) and various containers for holding and measuring liquids. We used three liquids: oil, water and mercury. Students never handled the mercury, but were allowed to lift a securely contained and wrapped amount of it (one pound) during one of the teaching sessions.

Organization of Class Sessions. The following is an account of the class sessions held after the intial interviews:

(1) First class (Introduction to the Computer Program): During the first class students became acquainted with the " Modeling with Dots" program, following the worksheet entitled "How to Use This Program" (see Appendix). When they finished this first sheet, they were given another which had a screen-dump picture of 3 objects. They were asked to construct these objects on their screens and then to order them by "size," "total number of dots," and "dots per size unit." They were then to build 3 more objects, this time getting the specifications from a screen-dump of the data.

(2) Second class (Using the Program to Order and to Model): In the second class, we reviewed the commands and the meaning of the data. We discussed the two ways the word "dots" could be used - clarifying the difference between "total number of dots" and "dots per unit size." We then had a discussion about what it means to order. We found several ways to order the members of the class as examples. (e.g., height, weight, age). Students were then asked to complete a worksheet that had ordering tasks based on computer drawn objects. Answers were discussed and put on the board. The next activity was to model groups of pennies and groups of beads with the computer. Simple intensive quantity word problems were given and students constructed solutions on the screen. We then gave students steel and aluminum cylinders of equal size, and told them how much they weighed (1 lb. and 1/3 lb.). We let them examine the cylinders and then had them represent the cylinders on the computer screen.

(3) Third class (Discussion of Modeling): After looking over the students' drawings of copied screen images, we made posters which typified

their ways of representing the groups of beads and pennies. The posters were taped to the blackboard and we discussed how information was represented. We articulated the "code" used and what we did and didn't include in the representation. Students realized it was important to be accurate and consistent in the mode of representing. We then broke the class up into 4 groups and handed each group a different map of the Boston area. One was a subway map, one a road map of Boston and surrounding suburbs, one a streetmap of the city, and one a souvenir map with drawings of Boston's buildings and boats in the harbor. Each group was to report on what was represented and the way it was represented. We concluded that one map was not better than another, but that each was consistent and served a different purpose.

(4) Fourth Class (Modeling Real Materials): We discussed the language of the computer and how to represent real objects, including the issue of size vs. shape. We then presented, discussed, and tried out a step by step way of modeling the size, density, and weight of real objects. We started with individual cubes of different materials, progressed to groups of cubes, and finally to solid cylinders.

(5) Fifth Class (Review): We reviewed the code used to represent material objects. Posters were made to compare the computer representations of size, weight and density (heaviness of material) to those used by students in the pre-interview. We concluded with some discussion of the relationships of the three quantities and some problems were given on the blackboard for students to try.

(6) Sixth Class (Small Group Sessions): We worked with two students at a time. Students were to select from a range of real materials, the ones which corresponded to pictures on the screen. We discussed some ways of extending the model, i.e. increasing the number of dots/size unit necessary to represent a certain material. Paper and pencil were used here as well. Attention was paid to representing quantitative relationships of the densities of several materials accurately. Several samples of differently tinted water were used to demonstrate the idea of intensity of color, and to relate this notion to the density of materials. Other analogies or examples of intensive quantities were generated (e.g., price, sweetness).

(7) Seventh Class (Sink and Float, part 1): There was a demonstration and discussion about ordering objects according to the density of their materials and whether we could order liquids according to their density. Emphasis was placed on developing a procedure for finding relative densities: take equal size portions of materials and weigh them. The heavier portion will be made of denser material. Later in working with mercury and steel we considered an alternative procedure: take portions of two materials which are equal in weight and compare their sizes. The smaller object is made of the denser material. Using these procedures, we established a density ordering for the following materials: brass, steel, aluminum, wood, oil, water, mercury. Many students thought that oil is denser than water because it is thicker. Weighing equal size portions of these liquids proved water to be denser than oil. The high density of mercury showed that solids are not always denser than liquids. The Archimedes program was then introduced. A brief demonstration was given in

front of the class and then students were allowed to experiment and play with the program for about 10 minutes. The object of this session was for students to come up with a rule that states when an object will sink and when it will float.

(8) Eighth Class (Sink and Float, part 2): We had a short discussion on the meaning of making a general rule based on observations and experiments. We presented some of the rule ideas generated by the class during the previous session (They had written these down.) We included a demonstration that color and weight were not generally good criteria for deciding whether an object will sink or float. Students were given another worksheet and instructed to restrict their investigations using the Archimedes program to finding cases of sinking objects; we asked them to come up with a rule about sinking objects. We went over the worksheets and discovered that density or dots/size unit of the liquid compared to the object was the relevant factor. The Sink the Raft program was then installed on the computers and students were given another worksheet. The main purpose of this part of the lesson was to find out if the size (and thereby the weight) of an object influences whether or not it will sink or float. We concluded the class by noting that neither size or weight are crucial factors; rather it is the density of the object compared to the liquid that is crucial. A couple of final puzzles were posed: 1) Why do balloons filled with helium float? 2) Here is a large piece of clay. It sinks. Here are two small equal size pieces of clay. One sinks and one floats. Is one a fake? Why?

The Post-interview

The post-interview differed from the pre-interview only in the following ways:

(1) In the post-interview, it was assumed that all students were familiar with the word "density." Hence, students were not asked "Have you heard of density?" Further, when we wanted them to order by density, we simply said, "Order these by the density of their materials" instead of by the "heaviness of their materials, that is the density of their materials." Finally, after they had finished the ordering tasks, we asked, "Do you think there is a difference between weight and density? What is the difference?"

(2) During the modeling task, in addition to their spontaneous models, students were asked to produce another drawing of the five objects using the computer notation.

(3) Students were shown a drawing which depicted a modified version of the computer model. That is, in this picture, dots stood for empty spaces so that steel was represented with 1 dot per block while wax had 5 dots per block. Students were asked to react to this model. ("...Useful? Like it? Good? Can you imagine the small spaces?")

(4) Finally, following the interview, students were queried informally about their reactions to the teaching sessions and the computer programs.

They were asked to comment frankly on what the liked best and least. We also invited their suggestions.

Results

Results are presented below for each of the two goals of our teaching intervention: (1) to help students create a concept of density which is distinct from weight and volume and (2) to help build students' metaconceptual awareness of modeling as a tool of science. At present most of our teaching efforts were targeted at the former goal and most of our questions in the pre- and post-interview assessed the degree of our success in achieving this goal. Hence, we will report most extensively on changes in students' conceptualization of density. However, the interview also bears on children's spontaneous modeling abilities, which would be important to understand in designing a curriculum to build greater metaconceptual awareness of modeling.

Students' understanding of density

There were three main contexts in which we assessed children's understanding of density in the individual interviews: (1) in questioning them about why same size objects made of different materials did not weigh the same and about why objects could weigh the same even though they were different sizes and made of different materials; (2) in requiring them to order a set of objects by weight, size, and density; and (3) in probing their understanding of the phenomena of sinking and floating. Within each context, children were questioned in a variety of ways: they answered direct questions, they were asked to do something (weigh objects of different sizes and made of different materials, sort and order objects, put objects in water to see how they behave), explain what they did, make predictions, formulate general rules or definitions, and make a model which expressed their ideas. For each task within a problem context, we analyzed children's pattern of responding and categorized children's patterns in two kinds of ways: first according the the specific rule or strategy the child used for the task (e.g., a rule based on weight, on heaviness of kind of material, etc.) and then according to whether such a rule showed no understanding of density, partial understanding of density or a clear understanding of density. In making these categorizations, all the data were independently scored by both a psychologist and a physicist. In general, we agreed on both types of categorization; any disagreements were discussed until we reached concensus. We also looked at children's patterns of understanding within an entire context, and across contexts, to get a sense of the larger patterns of change within children's thinking.

Questions about the factors that affect the weight of objects

Children's understanding of the weight of objects. There were three questions that probed children's understanding of some of the factors that affect the weight of objects: (1) the question about why two pieces of steel (one big and one small) did not weigh the same; (2) the question about why three same size objects (one made of wax, one made of aluminum, and one made of steel) did not weigh the same; and (3) the question about why objects made of different materials and of different sizes (a large aluminum cylinder and a much smaller steel cylinder) weighed the same.

In the pre-interview, all the children showed that they understood that the size of an object affected its weight. They said that one steel piece weighed more than the other because it was bigger, taller, etc. Indeed, most also commented that because it was bigger it had more material in it.

In the pre-interview, all the children also showed that they thought the kind of material of which an object was made affected its weight. None was surprised that same size objects could have different weights--they felt that they weighed different amounts because they were made of different materials and different materials had different weights. However, children were not very clear about the aspect of the materials that was different. A few children could be no more specific than to say the materials were somehow different. Because of the vagueness of their remarks, these children were categorized as showing no ability to articulate a concept of density (see Table 1). The majority of the children were able to say that some materials are heavier kinds than others. They went on to explain this fact in a variety of ways: some materials may be solid while others are empty or liquid, some materials may be stronger, and some materials may have more material in them because they are thicker. Further, these children did not expect heavier kinds of materials to always be heavier. They were not puzzled or surprised, for example, to learn that the large aluminum weighed the same as the smaller piece of steel, and could explain that although the piece of aluminum was larger, the smaller piece was made of a heavier material so they could add up to be the same. Taken together, these data suggest that this group of children had at least begun to develop two different senses of weight, and to distinguish between heavy objects and heavy materials. Thus, they were credited with a partial ability to articulate a concept of density. Finally, two children explicitly said materials differ in their density, and were credited with a clear articulation of the notion of density. One of these children suggested this meant their atoms are more closely packed.

The major change in the post-interview was that more children could clearly articulate the density of materials as a factor affecting the weight of objects (see Table 1). Now the majority of the children either explicitly said it was the density of the materials that was relevant (with most children accompanying their use of the word density with talk of the material being more packed or having more dots per size unit) or explained the relevant variable as how packed the different materials were. These children were credited with a clear articulation of density. The rest of the children talked simply about some materials being heavier kinds or being different. None of the children who persisted in talking about materials as being heavier and lighter kinds continued to explain these differences as resulting from the material being hollow/full, solid/liquid, or stronger/weaker, although some did persist in explaining the weight differences of materials in terms of the thickness of the material. Significantly, the latter explanation can be (on some interpretations) compatible with explanations in terms of packedness, while the former types of explanations are not.

Table 1

Changes in children's ability to articulate that the density of materials is one factor which affects an object's weight

	Post-interview		
Pre-interview	None (1)	Partial (6)	Clear (11)
None (3)	1	1	1
Partial (13)		5	8
Clear (2)			2

Children's representations of objects: Spontaneous models. After children were asked to explain why different objects weighed what they did (five objects: large and small steel; three objects the same size as the large steel made of steel, aluminum, and wax; and a larger aluminum object which weighed the same as the smaller steel), they were asked to represent what they had been talking about using a picture code. It was emphasized that it was not important that the picture be realistic, just that it communicated in some fashion the ideas they had been talking about. Children were given some paper and colored pencils to work with. After they had completed their drawing they were asked to explain what they had represented in their drawing and to describe how they represented it. Of particular interest to us here is what attributes of the objects children choose to represent (in a later section we will explore the types of codes used and the consistency with which these codes were used). This task thus allows us to judge whether children have a way of visually representing the density differences of materials.

In the pre-interview, most of the children attempted to represent the differences in size and weight of the objects (see Table 2). However, fewer children said they represented the materials the objects were made of, and only 2 of the children explicitly said they depicted the heaviness or density of materials in their drawings. Instead, five children said they represented the color of the objects as a relevant variable (this response was not counted as a representation of material).

Most typically, outlines of whole objects were shown of varying heights to indicate size differences (the objects were all cylinders with a common diameter). Children indicated by numbers what the weight of the whole object was (they had weighed the objects on a postage scale). And children indicated by using different colored outlines for the whole object what color or material they were. For these children, then, the weight of the material is not explicitly depicted as a local or separate property from the weight of the whole object. Thus, although most children talked about heavier kinds of materials in the questions preceding the modeling task, most did not know how to represent this notion in a model.

Of the two children who did attempt to depict the heaviness or density of the materials in the pre-interview, one said he was depicting the heaviness of the material (and did not separately represent weight) while the other said he was representing the density of the material (and did have a separate representation of weight). Both adopted a similar representation for the density of materials: they filled the objects with varying shades of color (ranging from lighter to darker;); the darker color stood for the heavier or denser material. The child who talked of density used shades of gray which stood for how packed the atoms were. The child who talked of the heaviness of materials used layers of color: the steel was purple and the wax was yellow, while the intermediate aluminum was purple streaked with yellow.

Finally, it should be noted that children varied in the total number of relevant dimensions they attempted to represent. The maximum number of relevant dimensions to represent was four (size, weight, material, and

Table 2

Children's Models:
Changes in what dimensions children attempt to represent from the pre-interview to post-interview

Dimension	Pre-interview Spontaneous Model	Post-interview Spontaneous Model	Post-interview Computer Model
Size	15	18	16
Weight	12	13	12
Material	8	14	11
Density	2	8	13

density). In the pre-interview, children typically represented one, two or three (see Table 3).

By the post-interview, more children were explicitly representing the material kinds and the densities of the materials than in the pre-interview, although the numbers representing size and weight remained the same (see Table 2). In representing the heaviness or density of materials in the post-interview, 3 used shades of color to stand for increasing density (as used in the pre-interview), 3 used dots/size units, 1 used words, and 1 incorrectly used inverse order of size. Overall, 6 children moved from making no attempt to represent the density of the material to making such an attempt, with five of these six now attempting to represent both weight and density in their models. Table 3 shows that children in the post-interview also attempted to represent more relevant dimensions in their model. Now children ranged from representing two to four dimensions, with nine children increasing the number of dimensions represented from the pre-interview, and six children attempting to represent all four dimensions.

Thus, prior to teaching, most children did not spontaneously attempt to represent either the heaviness or density of materials in their models; indeed, many did not even spontaneously represent the different kinds of materials. Teaching resulted in more children being able to do so. At the same time, it should be noted that many still did not represent density and only four used variants of the computer model in their spontaneous models.

Children's represention<u></u>s of objects: Computer models. In the post-interview we also asked children to draw a model of the five objects using the computer model. They were asked to draw a representation of the five objects on paper, using the notation of the computer model, not literally to model the objects using the computer. These instructions brought about an even greater attempt to represent the density of materials. As Table 2 reveals, now most children attempted to represent the density of the materials. (All but one used the standard convention of dots per size unit; the other used an invented code of number of squares per row.) Further, these children (with two exceptions) consistently expressed the important features about the densities of the five objects: correctly portraying the two steel objects as having the same density despite their difference in size and weight, correctly portraying the two different size aluminum objects as having the same density, and correctly showing the wax to be less dense than the aluminum and the aluminum to be less dense than the steel. than the steel. The two children who were exceptions were able to achieve local consistency in expressing density relations: among the same size objects, they correctly portrayed steel as denser than aluminum and aluminum as denser than wax, and among the equal weight objects, they correctly portrayed the small steel object as being made of a denser material than the larger aluminum object. However, they did not show the two aluminum objects to be made of material of the same density, or show the two steel objects to be made of material of the same density. Only one child worried about the exact quantitative relations among the objects in depicting their densities. He said that steel was three times denser than aluminum, and aluminum was three times denser than wax, and noted he couldn't represent all three using the limited types of material (1 to 5 dots/size unit) of the computer model. The other children were only concerned with showing that steel was denser than aluminum and aluminum was denser than wax, and picked specific numbers for their densities more arbitrarily.

A few of the children made no explicit attempt to represent the densities of the materials. Instead, they simply tried to represent the weights of the objects. Significantly, some of these children erroneously used dots/size unit as a representation of weight. In addition, one child who consistently used number of squares per row as a representation of density, then chose to represent weight as number of dots per size unit.

In the post-interview, children attempted to represent approximately the same number of dimensions in both their spontaneous models and their models using the computer notation (see Table 3). However, children were more likely to represent density when instructed to use the computer notation (see Table 2). Significantly, even some children who represented only one or two dimensions, chose to represent the density of the materials when using the computer notation. This never occurred in their spontaneous models where density was represented only by those attempting three or four dimensions. Thus the computer notation seems to make the dimension of density more salient to children.

<u>Summary</u>. In the pre-interview, most children used the expression "heavier kind of material", which may label a precursor density concept. However, they did not spontaneously represent this quantity when asked to construct models. By the post-interview, approximately two-thirds of the children now had separate language for talking about density and weight and could accurately portray some qualitative information about the densities of materials when instructed to use the computer model. Children were also more likely to represent density in their spontaneous models, although the sophistication of their spontaneous models in this regard lagged behind their skill in using the computer models. At no point did students simply incorporate the computer model wholesale; instead they assimilated it to their own beliefs, often modifying or adapting it in unique ways.

<u>The ordering tasks</u>

<u>Children's understanding of the word "density".</u> A first question concerns whether children had heard of the word "density" prior to the pre-interview and could explain what it meant. We found that 10 of the 18 children had never heard of the word "density" and had no idea what it meant. Four children had heard of it but thought it referred to the object's weight, size or shape. Both groups of children were categorized as having no understanding of the word "density" or of the difference between density and weight (see Table 4). Thus, the majority of the children did not know or correctly understand the word "density" in the pre-interview. A few children gave evidence of having some partial understanding of density--two said it had to do with whether something sinks or floats, and one said it had to do with what a substance contains. Finally, one student was credited with having a clear understanding of density (at least at a beginning level): he said density referred to how packed a substance is .

Table 3

Children's Models:
Changes in the number of relevant dimensions children attempted to represent from the pre- to post-interview

Number of Dimensions	Pre-interview Spontaneous Model	Post-interview Spontaneous Model	Post-interview Computer Model
1	6	0	2
2	5	7	5
3	6	5	4
4	1	6	7

Table 4

Changes in children's understanding of the word "density" between the pre-interview and the post-interview

	Post-interview		
Pre-interview	None (7)	Partial (2)	Clear (9)
None (14)	7	1	6
Partial (3)		1	2
Clear (1)			1

By the post-interview, we knew that children had heard of the word "density" since we used it extensively in the teaching sessions. Thus, instead of asking them "Have you heard of density? What is density?" we rephrased the question as "Do you think there is a difference between weight and density? If so, what is it?" Now half of the students could clearly articulate a correct difference between weight and density (see Table 4). They expressed their insight that weight was a property of the whole object while density was a local property in a variety of ways: weight refers to the whole thing while density refers to a part; weight refers to the total number of dots while density refers to the number of dots per size unit; the size of an object affects its weight, but not its density; weight refers to how much something weighs while density refers to the weight per size unit or how packed something is. A few students thought of weight and density as different, but had only a partial understanding of density. They both said that smaller objects were denser (and more packed) than larger objects, but could be lighter. These children were credited with having only a partial understanding of the difference between weight and density, because they did not articulate the part/whole distinction. Finally, the rest of the children still could not articulate a difference between weight and density. It should be noted, however, that some of themthought there was a difference although they could not articulate it, while the others explicitly said there was no difference.

Table 4 thus shows the change in children's ability to articulate what density is and how it differs from weight between the pre- and post-interview. Whereas only 1 child could clearly explain what "density" meant in the pre-interview, half the children could do so by the time of the post-interview.

Ordering the cubes. Children were asked to order a set of five objects by their weight and by the density of the material each object was made of. The objects were made of varying materials and varying numbers of 1 cm cubes arranged in different shapes. The set of objects was selected so that an ordering by density was quite different from an ordering by weight. In particular, there were three objects made of aluminum: a very light aluminum piece and two much heavier aluminum pieces. In a weight ordering these pieces would be put at almost opposite ends of the order, while they would be grouped together in a density order. In addition, there was a small copper piece which was lighter than the larger steel object. Thus, the copper object would be placed before the steel object in a weight ordering, but after the steel object in ordering of the density of the materials. A balance scale was available so that children could compare the weights of objects or individual cubes if they wished; indeed they could manipulate the objects in any way they desired to help them with the task.

Because we could not assume that children knew the word density in the pre-interview we introduced the density ordering task in the following way. We first had children sort the objects by the material they were made of. Any errors in identifying materials were corrected at this time and the names for the four different materials were introduced. Although some children initially made some errors in sorting by material (they were not always sure that the small alluminum or steel cubes were made of the same material as the large cylinders), all children seemed to readily understand this part and any corrections we made. Then we asked children to order the objects "according to the heaviness of the kind of material they are made of, that is according to the density of the material." In the post-interview, this phrasing was not necessary and we simply asked them so order the objects by the density of the material they were made of.

The critical question was to what extent children ordered the objects in different, relevant ways when asked to order by density than they had when asked to order by weight. Most all of the children were able to order the objects by weight in both the pre and post-interview when the instructions were to order by weight, and articulated a relevant procedure for determining the weights of objects (i.e. lifting whole objects in separate hands and feeling the differences, putting two objects on a balance scale and comparing the differences). There were some errors in their weight orderings, but these errors seemed attributable either to their relying on felt weight (and being subject to, for example, the size/weight illusion for particular items) or their forgetting to check a particular comparison when inserting an object into the order (i.e., not being completely systematic in their procedure for ordering), rather than their misunderstanding what weight was. We thus used their ordering produced when the instructions were to order by weight as a baseline for interpreting the ordering they produced when the instructions were to order by density.

In the pre-interview, half the children simply ordered the objects in the same way they had with simple "weight" instructions, and are categorized as showing no understanding of density as a distinct quantity (see Table 5). One-third of the children showed the insight that all the aluminum pieces should be grouped together regardless of weight when asked to order the objects by density. However, these children failed to order the four groups of materials correctly by density. The most common error was to judge copper to be less dense than steel, because the small copper piece was lighter than the larger steel piece. These children were credited with a partial understanding of density since they seemed to realize that objects made of the same material have the same density, but they did not yet have a systematic procedure for determining which objects are denser than others (e.g., compare one cube of copper to one same size cube of steel). Only a few children were able to order the materials correctly by density: rubber, aluminum, steel, copper. These children not only put all the aluminum objects together in their order, but also realized that copper was a denser material than steel even though they had judged the copper object to be a lighter object than the steel object in their weight ordering.

Table 5 also shows the progress children made in the density ordering task by the post-interview. Now the least typical response was ordering by weight, and the most typical response was ordering by density . Ten of the 15 children who did not order by density in the pre-interview made some progress in their ordering: 4 progressing to a partial understanding of density (grouping material kinds together, although not ordering the kinds completely correctly) and 6 progressing to ordering by density. Not

Table 5

Changes in children's ability to order the cubes by density between the pre-interview and post-interview

	Post-interview		
Pre-interview	None (3)	Partial (6)	Clear (9)
None (9)	3	4	2
Partial (6)		2	4
Clear (3)			3

surprisingly, many of those who initially ordered by weight moved to grouping materials together, while those who were already grouping like materials together progressed to a full ordering by density.

Students' descriptions of their strategies in ordering the cubes. Immediately after children ordered the first set of objects, they were asked to explain how they had known how to order them. In general, their verbal explanation of their strategy was consistent with the strategy we had inferred on the basis of their ordering and supports the categorization of the children into three groups: those who use the same strategy for the weight and density questions, those who use a different, partially correct strategy for ordering by density, and those who use a different and fully correct strategy for ordering by density.

Table 6 shows how children's explanations changed from the pre- to post-interview. In the pre-interview, the dominant strategy for the density task was to weigh whole objects on the balance scale or compare their weights proprioceptively. These children thus articulated the same strategy for the weight and density tasks, and are categorized as showing no understanding of density. Most of the rest of the children said they could tell by looking at the materials, or by putting together the materials with the same name. Since they implied that objects made of the same materials had the same density, they were credited with a partial understanding of density. However, they did not articulate an explicit procedure for ordering by density: such as, comparing the weights of equal size pieces. Finally, one child was able to articulate such an explicit procedure for determining the density of materials, and was thus categorized as having a clear understanding of density. By the post-interview many more children articulated a clear strategy for inferring relative densities (see Table 6) and the majority of children at least articulated a partially correct strategy for determining densities.

In general, there was a strong relation between children's pattern inferred from their behavior in ordering and their explicit explanation of their ordering. Twelve out of 18 children gave an explanation consistent with their inferred pattern in the pre-interview; 13 out of 18 in the post-interview. In 10 of the 11 cases where there was a mismatch, their explanation of their strategy was 1-step below their inferred pattern (children who ordered by material, simply referred to the weights of objects; children who ordered by density, simply referred to the materials). Thus, it seems children's ability to use a strategy may precede their ability to verbalize it.

Ordering the cylinders. After children had ordered the first set of objects, they were presented with three new objects (a small steel cylinder, a slightly taller aluminum cylinder, and a very tall aluminum cylinder) and asked to insert these objects into the order they had just produced. When ordering by weight, all three objects come at the end of the order because they are clearly much heavier than the objects made of cubes, and the small steel cylinder is equal in weight to the larger aluminum cylinder. However, when ordering by the density of materials, these objects need to be placed with the objects made of the same materials in the earlier orders. This portion of the task thus tests how strongly

Table 6

Changes in children's ability to describe a distinct strategy for ordering by density between the pre-interview and post-interview

	Post-interview		
Pre-interview	None (6)	Partial (5)	Clear (7)
None (11)	5	3	3
Partial (6)		2	4
Clear (1)	1		

children believe that objects made of the same materials have the same density. Because the objects cannot be decomposed into little cubes there is no way they can directly test or compare these cylinders with the other objects.

In the pre-interview, children overwhelmingly ordered the cylinders by weight for both the weight and density ordering tasks; thus the majority of children are categorized as showing no understanding of density on this subtask in the pre-interview (see Table 7). Only 4 children put the cylinders with cubes made of the same material. Since 3 of these 4 children had ordered the cubes simply on the basis of kind of material (and not density) it is likely that their success reflects simply a strategy to put like materials together rather than an ability to imagine that an equal size piece of the cylinder would weigh the same as a small piece of the cube. They are thus credited with only a partial understanding of density. Only the one child who both sorted the cubes by density and placed the cylinders with like cubes made of like materials is credited with a clear understanding of density.

By the post-interview, the majority of children showed at least a partial understanding of density in their ordering of the cylinders. Seven not only sorted the cubes by density but put the cylinders with their respective material kinds. Four other children had also progressed to showing a partial understanding of density in this task: two initially started to order by weight but then when the experimenter reminded them of the question they were able to think their way through to the correct answer; and two who had formulated a partially correct understanding of density in the cubes task (the smaller objects are denser because they are more packed) proceeded to apply this rule to the cylinders as well.

Summary of the ordering tasks. In all, there were four ways children's understanding of the difference between weight and density were probed in the ordering tasks: (1) asking children to order the first set of objects by weight and density; (2) asking children to explain how they had ordered them; (3) asking children to insert three cylinders into the order; and (4) asking children to explain the meaning of density (in the pre-interview) and to explain the difference between weight and density (in the post-interview). Looking at children's answers to these questions as a whole allows us to see some of the ways their understanding of density changed from the pre- to post-interview.

In the pre-interview, the majority of children (12) showed no understanding of density in the ordering tasks. They ordered the cubes and cylinders essentially by weight with both density and weight instructions, explained their strategy for ordering solely in terms of weight, and showed no understanding of the meaning of the word "density". Some children (5) showed a partial understanding of density in the ordering tasks: they ordered the cubes and cylinders consistently by material, or vacillated between ordering the cubes by density and the cylinders by material. Given that they did not have a clear understanding of the word "density", the locution "heavier kinds of material" was sufficient to at least focus them on materials. Only one child clearly [illegible]erstood the word "density". He also correctly ordered both the c[illegible] and cylinders by density and

Table 7

Changes in children's ability to order the cylinders as well as the cubes by density between the pre-interviewand post-interview

	Post-interview		
Pre-interview	None (6)	Partial (6)	Clear (6)
None (14)	6	4	4
Partial (3)		2	1
Clear (1)			1

articulated a correct strategy for inferring the relative densities of materials.

By the post-interview, half the children showed a fairly clear understanding of density by ordering the cubes according to density and then either explicitly articulating their strategy and clearly explaining the difference between weight and density or correctly inserting the cylinders into the order. Some other children now had a partial understanding of density: two could clearly articulate the difference between weight and density but did not fully apply this understanding to the ordering; and two had formulated an explicit (but incomplete) understanding of density which they consistently applied in the ordering tasks (they knew denser materials were more packed and incorrectly assumed smaller objects were therefore denser). Thus, in contrast with the pre-interview where the majority of children revealed no understanding of density in the ordering tasks, the majority now had at least a partial understanding of density in these tasks.

<u>The sinking and floating tasks</u>.

<u>Children's ability to formulate a general rule about what sinks and what floats.</u> Children were first given eight objects to see how they behaved in water. The objects were made of four different materials: two materials that were denser than water and two materials that were less dense than water. The objects were also of varying sizes, so that for each type of material, one object made of that material was heavy and one was light. After trying each object in the water and noting which ones sank and which ones floated, children were asked: "What kinds of things sink and what kinds of things float?" "Can you make a general rule that will allow us to predict what things will sink and what things will float?"

In the pre-interview, all but three children attempted to formulate a rule for why things sink and float. These rules were of two general types: (1) rules based on weight (heavy things sink and light things float) and (2) rules based on kind of material (heavy materials sink and lighter materials float). Children who were unable to formulate any rule or who only came up with a rule based on weight were categorized as not yet understanding the role of density in sinking and floating, while the children who expressed the rule in terms of kind of material were credited with a partial understanding. No child formulated a rule strictly in terms of density, although three mentioned density along with the factor of weight. These children were credited with having only a partial understanding because they had not yet focused on density as the sole integrating variable. Thus, overall in the pre-interview children were fairly evenly split between having no understanding of the role of density in sinking and floating and having a partial understanding (see Table 8).

Table 8 shows that, by the post-interview, children's rules for sinking and floating had become more sophisticated. Now only a few children could not formulate any rule or formulated a rule only in terms of weight (the no understanding of density category). The rest formulated a rule either based on heaviness of kind of material or explicitly in terms of density. Among the children who focused explicitly on density as the

Table 8

Changes in children's ability to formulate a general rule for sinking and floating between the pre-interview and the post-interview

	Post-interview		
Pre-interview	None (4)	Partial (7)	Clear (7)
None (10)	4	5	1
Partial (8)		2	6
Clear (0)			

key factor, three simply said that dense materials sink and less dense materials float, while four fully explained that materials denser than water sank and less dense than water floated. Thus, the majority of children clearly improved in their ability to state a general rule about sinking and floating between the pretest and posttest.

Predictions about the sinking and floating of objects. Children might, of course, have correct intuitions about what types of things would sink and float without being able to formulate a general rule verbally. Thus, a different way to assess their understanding of sinking and floating is to show them one object made of a certain material (which sinks or floats) and then ask them to predict whether another object made of the same material, but radically different size, would sink or float. Children were shown a small piece of wax which floated and were then asked whether a large wax piece would sink or float and to explain their prediction. Similarly, they were shown a medium sized aluminum cylinder which sank and were then asked to predict whether a small aluminum paper clip would sink or float and to explain how they knew.

Table 9 shows children's ability to predict whether the large wax piece and small aluminum paper clip would sink or float and to explain their prediction by invoking the idea that the two wax (or aluminum) pieces were made of the same materials (and/or had the same density). Again, there was a shift from the pre-interview to post-interview in the dominant category of response. In the pre-interview a large group of children made at least one incorrect prediction and gave at least one justification in terms of the weight of the object (the large wax object will sink because its heavy; the small aluminum object will float because its light). These children clearly did not even have the correct intuitions about the problem, and were categorized as having no understanding of the role of density in sinking and floating. A second group of children made correct predictions but could not explain their predictions in terms of sameness of material or density. Thus, they correctly predicted that both the large wax object would float and the paper clip would sink, but then explained their predictions by invoking the weight of the object, or the fact that the paper clip had holes in it, or offered no explanation at all. Because their correct predictions were not accompanied by a clear explanation in terms of sameness in density or material, they were credited with only a partial understanding of the relevance of density in sinking and floating. Only one child in the pre-interview was able to give both predictions and explanations which indicated she clearly understood the relevance of density. By the post-interview, half the children now gave clear predictions and explanations of their predictions using the notion of common material or density.

Children's descriptions of why an object sinks in one liquid and floats in another. The final phenomenon children were shown was that a piece of lucite floats in one liquid (salt water, colored with blue food coloring) and sinks in another (plain water, colored with red food coloring). Children were asked to explain how this could be.

In the pre-interview, many children had no idea why this could be, talked loosely about there being different chemicals in the water, or had

Table 9

Changes in children's ability to predict and explain the sinking and floating of wax and aluminum between the pre-interview and the post-interview

	Post-interview		
Pre-interview	None (8)	Partial (1)	Clear (9)
None (14)	7	0	7
Partial (3)	1	1	1
Clear (1)			1

the wrong intuitions about the phenomena (i.e., they talked in terms of the thickness/thinness of the water, but then argued the water in which the object sank must be thicker because it exerted more force on the object to push it down). These children were categorized as not understanding of the role of density in this situation (see Table 10). The rest of the children also gave intuitive answers, but their intuitions were basically correct. That is, they talked in terms of the thickness or thinness of the liquid (or the amount of air in it, or its strength), and then argued that thicker liquids could support objects that thinner liquids could not. These children were categorized as having a partial understanding of the role of the relative densities of objects and liquids in sinking and floating. No child in the pre-interview explicitly discussed the situation in terms of relative densities.

Table 10 shows that by the post-interview a number of children had increased their understanding of this situation: some progressed to having clear intuitions about the situation while others moved to being able to talk about the phenomenon in terms of relative densities.

Summary of sinking and floating tasks. Children's understanding of the role of density in sinking and floating was assessed in three ways: (1) by their ability to formulate a general rule governing sinking and floating; (2) by their ability to predict and explain whether wax and aluminum would sink or float, using the idea of same material and/or density; and (3) by their ability to explain why an object sank in one liquid but floated in another. Again, we looked at individual children's patterns of responding across these questions to see how well they understood the phenomena of sinking and floating, and in what ways their understanding developed from the pre-interview to the post-interview.

In the pre-interview, children were split into two main groups: those who had no intuitions about the role of density and materials in sinking and floating (7 children), and those who may have had some beginning intuitions (10 children). Children were categorized as having no intuitions about the role of density in sinking and floating if they were unable to formulate a rule which referred to material kinds and if they did not make correct predictions about the wax and aluminum objects. Children were categorized as having some beginning intuitions if they were able to either (1) refer to material kinds in their general rule about what things sink and float and give intuitive explanations about why the lucite could float in one liquid and sink in another; or (2) make correct predictions about whether the wax and aluminum would sink or float (without being able to consistently explain their predictions). At this stage, children's ability to appeal to material kinds in their general rule was highly correlated with their ability to have correct intuitions about why the lucite sinks or floats in the red and blue liquids. However, children's verbalizations were not predictive of their ability to give consistent predictions about the wax and aluminum. Only one child was able to show such consistency in the pre-interview.

By the post-interview, half the children showed consistency in understanding of sinking and floating. Not only could these children correctly predict whether the wax and aluminum would sink and float, they

Table 10

Changes in children's ability to explain why the lucite floats in the blue liquid but not the red liquid between the pre-interview and the post-interview

	Post-interview		
Pre-interview	None (4)	Partial (8)	Clear (6)
None (8)	3	3	2
Partial (10)	1	5	4
Clear (0)			

could explain their predictions in terms of the materials or densities, and had formulated a general rule consistent with their predictions. Seven of these nine children also used the word density in their verbal formulations, while the others talked simply in terms of heavier kinds of materials.

Summary: children's understanding of density

Overall, children were elaborating a distinct concept of density as a result of the teaching in a variety of ways: learning a richer model for representing density and a new language for talking about density, learning how to order objects by relative densities, and learning how to apply a concept of density to predict the sinking and floating of objects. A final question concerns the interrelations among these developments: (1) to what extent was success at ordering dependent upon the child's correct assimilation of the computer model and acquiring an ability to verbalize the difference between weight and density? and (2) to what extent was success at understanding the role of density in sinking and floating dependent upon the child's success with ordering objects by density? Tables 11 and 12 show that performance on the various tasks was highly inter-related.

Consider first the relation between ordering the cubes by density and being able to represent density correctly with the computer model or articulate the difference between weight and density (Table 11). Understanding the difference between weight and density (as reflected by proper depiction of density using the computer model or verbal articulation of the differences between weight and density) appears to be necessary but not sufficient for successful ordering of the cubes by density. Every child who was successful at ordering the cubes by density gave evidence of understanding the distinction between weight and density in one of these two ways. However, a few children who gave evidence of such understanding, still failed to order correctly. The rest of the children who had difficulty with ordering had given no evidence ofa basic understanding of the distinction between weight and density.

Table 12 also shows there is a relation between being able to order the cubes by density and being able to articulate a rule for sinking and floating explicitly in terms of density and then use this rule to make correct predictions about wax and aluminum. Six of the seven children who formulated a rule for sinking and floating in terms of density also successfully ordered the cubes by density; but there were a number who successfully ordered the cubes by density who did not apply this understanding to sinking and floating. Thus, having a clear concept of density may be necessary but not sufficient to ensure application to the area of sinking and floating.

Children's understanding of modeling

Ultimately, we are interested in developing children's meta-conceptual understanding of modeling as a tool of science and of criteria for evaluating good models. In such teaching, we are interested in conveying to children that models can be abstract (depicting ideas, and not

Table 11

The relationship between children's ability to order the cubes by density and their understanding of density as an intensive quantity (post-interview only)

Understanding of the intensive nature of density	Ordering of cubes	
	Use density	Do not use density
Articulate explicit difference between weight and density and/or represent density correctly with computer model	9	3
Do not show such understanding	0	6

Table 12

The relationship between children's ability to formulate and use density in a rule about sinking and floating and their ability to order the cubes by density (post-interview only)

	Sinking and floating	
Ordering cubes	Use density	Do not use density
Use density	6	3
Do not use density	1	8

necessarily concrete objects); that good models need to be consistent and accurate, and quantitative where appropriate; and finally that models should be evaluated for their usefulness for specific purposes and not for underlying truth. At this point, however, we only began to broach these subjects with students in our teaching; and through our individual interviews we were able to assess only the extent to which student's may have understood these points intuitively while constructing their own spontaneous model.

In an earlier section, we reported what dimensions children attempted to represent in their spontaneous models and noted that children increased in the number of dimensions they represented and in their likelihood of representing density from the pre- to post-interview. In this section, however, we report on two aspects which bear on the overall quality of the representations: the consistency with which children were able to represent a particular dimension (for the five objects in question) and the sophistication of the type of code. Let us consider each in turn.

Consider first children's ability to represent a dimension consistently in their models. For each dimension that children attempted to represent, they were scored as being either partially or fully consistent in their representation. There were several ways the child could be credited with only partial consistency. First, sometimes the child represented a dimension for only a subset of items: for example, representing the weights for the three same size items, but then not representing the weights for the two equal weight items. Or, children might be consistent in their representation of a particular dimension only locally, but not across all five items. For example, the child might show the three same size items to be the same size, and the two equal weight items to differ in size, but not correctly show that the size relation between these two subsets of items. Or sometimes the choice of representation captured only some of the important properties of a dimension. In contrast, full consistency required that one be able to tell the relationship among all five items on the dimension in question.

Table 13 shows the degree to which children represent a dimension consistently in their pre- and post-interview spontaneous models and in their post-interview computer based models. The striking aspect of the results is that while more children are able to represent material and density in the post-interview models in a consistent fashion, children decrease in the consistency of their representations of size and weight in the post-interview. Of course, overall the number of dimensions consistently represented remains the same, while the number of dimensions attempted (albeit inconsistently) increases. Since children were attempting to represent more dimensions, they may have been too overloaded to represent them all consistently. Further, it may be an initial consequence of strengthening their density concept, that their size and weight concepts are correspondingly weakened. Since our teaching focused primarily on understanding density and our computer model makes density the most transparent quantity (with more calculation needed to represent size and weight correctly), we may simply need to pay more attention to the concepts of size and weight in our future teaching.

Table 13

Number of Children Showing Consistency in Representing Different Dimensions in the Pre- and Post-Interviews

Level of Consistency	Size			Weight			Material			Density		
	Pre	Post	Comp	Pr	Pst	C	Pr	Pst	C	Pr	Pst	C
Full	12	9	4	7	4	3	7	11	10	2	5	9
Partial	3	9	12	5	9	9	1	4	1	0	3	4
None	3	0	2	6	5	6	10	4	7	16	10	5

Consider next the type of picture code children used in their models. Children were scored as using one of four types of codes: verbal code (in which they describe a dimension in words), pictorial code (in which they represent a dimension as they see it), numeric code (in which they represent a dimension as a summary number), and symbolic (in which they represent a dimension in some abstract way). Table 14 shows the number of children using each type of code for each dimension. There is a big increase in the number of children using symbolic type codes from the pre-interview to the post-interview, with the number using the other types of codes remaining fairly constant. By the post-interview, this is the dominant type of code for each dimension. At the same time, there is some variation from dimension to dimension in type of code typically elicited (especially in the pre-interview). Size brings out a tendency to represent pictorially, with many children using perspective to depict the cylinder shape of the objects. These children may not have fully distinguished between a picture and a more abstract rendering of objects (two children even put in a representation of the circular tops of the objects in their drawing with the computer model--although the experience with the computer model had always been with squares). And weight seemed the one dimension that initially brought out use of numeric codes. Perhaps this reflects the fact that it is the only dimension which is easy to measure directly (put it on a scale and read a number). Measuring size and density is much more indirect.

Overall, however, it was clear that most children were comfortable with using some abstract and symbolic representations in their models. This was shown not only by the fact that they ignored the shape of the object in depicting the size, but also by the fact that when they used color codes for material, or heaviness, they frequently used colors that were different from the actual colors of the objects (e.g., they had green or blue stand for aluminum), or used the dots per size unit code for density.

Discussion and Conclusion

In general, our teaching strategy proved to be moderately successful with this group of 6th graders. Our aim was to help consolidate their understanding of the distinction between weight and density by helping them understand that weight was an extensive quantity and density an intensive one. We provided a visual model in which the quantities of size, weight, and density were all salient, to help them see that adding material to an object changed its size and weight but not its density, and gave them an explicit language for talking about densities in terms of the model. Further, we explicitly taught them a procedure for ordering objects by relative densities, embedding the teaching of this procedure with instruction in the basic model which would allow them to understand why this procedure makes sense. Finally, we involved students in experimenting with computer simulations of sinking and floating, and directed them towards extracting a predictive rule involving the relative densities of objects and liquids in understanding this phenomena. We found that the majority of children did correctly assimilate this model in a way that supported their understanding of density as an intensive quantity, and were able to articulate some relevant differences between weight and density.

Table 14

Number of Children Using a Particular Type of Code in
Representing Different Dimensions in the Pre- and Post-Interviews

Type of Code	Dimension							
	Size		Weight		Material		Density	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Symbolic	8	13	2	7	6	11	2	6
Numeric	0	0	8	5	0	0	0	1
Pictorial	9	7	0	0	1	2	0	0
Verbal	0	1	3	3	1	2	0	1

Further, there was evidence that distinguishing weight as an extensive quantity and density as an intensive quantity helped them to understand why in ordering objects by relative densities, it is necessary to compare equal size pieces. Only the children who had correctly internalized the difference between weight and density were able to remember the ordering procedure we had taught. Finally, a clear understanding of the distinction between weight and density was important in being able to apply such an understanding to the phenomena of sinking and floating.

At the same time, we found that not all the children were able to correctly assimilate the model, or to verbally articulate the difference between weight and density. Further, we suspect that many of the children who correctly assimilated the model were not yet able to deeply understand the theory underlying it (that is, they did not spontaneously extract the mathematical relations depicting the relations among the three quantities). Thus, it is important to consider what kinds of difficulties arise in assimilating/understanding the model and how these difficulties can be addressed in future teaching efforts.

There were two main types of errors children made in assimilating the model: (1) some seemed to remember only that number of size units represented size, and total number of dots represented total weight, ignoring how the model represented density; and (2) other children again focused only on the representations of size and weight (ignoring density), but these children incorrectly assumed number of dots per size unit was a representation of weight. We had thought that the three distinct variables in the computer model would be obvious to the children, and given that children have at least three distinct dimensions in their intuitive theory--size, weight, and heavy for size, they could make the mapping between these concepts and the model. Both types of errors, however, reveal children's failure to make any mapping between their precursor density concept and the model. There are at least two distinct explanations for this difficulty. Perhaps these children did not have a well enough developed precursor density concept to make even this initial mapping. Since we did not give an extensive battery of tasks designed to assess such an early concept (as did Smith, Carey, and Wiser), it is hard to test this possibility with our own data. But we suspect this is not the complete explanation since most of these children did talk of materials as being heavier kinds and were able to invoke a compensation argument to explain why the large aluminum object could equal the small steel object in weight. Another more plausible explanation is that the way that the computer model is introduced to children with teaching activities could be improved. In our teaching, modeling was introduced at the very beginning, with very little explanation or motivating context. It may be important that children are first introduced to activities which invoke their pre-existing concepts of size, weight, and density, and then a situation could be presented where modeling is seen as helping children solve some problem (see chapter 5 for a fuller discussion of what these changes in teaching approach might involve). This would ensure that more students were thinking about the modeling task in a conceptual manner rather than as an arbitrary jumble of symbols to be learned in a rote fashion. We suspect that those children who made the error of mapping dots per size unit with weight were simply approaching the mapping task in a superficial manner.

In the real world, weight is a more salient quantity than density since we can feel the weights of objects simply by hefting them. In the computer simulation, however, dots per size unit is more salient than total number of dots--both because it is more immediately quantifiable (without recourse to tedious counting) and because it is a variable the child can directly manipulate. Thus, the child may be simply mapping the two most salient variables without thinking deeply about underlying meaning. Providing a more meaningful context for doing the initial modeling activities may be enough to help these children understand the computer model correctly.

Our experience with teaching also helped us identify other places where our approach to teaching could be extended and improved. We found that even the children who correctly assimilated the model, spontaneously assimilated it only in a qualitative way. They were concerned with portraying which materials were denser than others, but were not yet concerned with issues about how much denser. This in turn led them to have problems with correctly representing weight (in particular, objects made of materials of different densities which were equal in weight), although most children were not too concerned with these problems. Indeed they represented the size and weight of objects in very rough and approximate ways. This is probably fine and appropriate for a beginning; indeed, we explicitly tried to build only a qualitative understanding of the model in our present teaching. However, ultimately, we would like them to exploit the model's quantitative potential and to think more precisely about all three quantities. Our experience suggests that explicit teaching activities will need to be developed to motivate students to see the relevance of greater precision, and to grasp the mathematical inter-relations among the quantities. This level of understanding of the model does not occur simply spontaneously.

Another area where the teaching unit should be expanded concerns the unit on sinking and floating. We suspect that the reason that not all children who were able to develop a concept of density could apply it in understanding sinking and floating was that we did not give them enough time to explore these phenomena. It was the shortest aspect of the teaching (2 sessions), even though it was one of the most naturally intriguing and motivating to the students. Indeed children uniformly reported that they liked this aspect of the whole teaching unit the best. Further, although there was much they did not understand about this situation, it was one of the few areas where they often had good initial intuitions. Thus, in the future, we plan to begin by posing some puzzles about sinking and floating before introducing the problem of modeling as a way of setting a context for those activities and allowing them to have more time to explore the phenomena both with real world materials and the computer program. Indeed, we can organize the whole teaching unit more centrally around these phenomena.

Finally, our teaching experience suggested some ways that we might expand the range of models presented to children for their consideration. One of the striking aspects of the data was that although the majority of children understood the model and could use it appropriately when explicitly asked to, few spontaneously chose to use the model when initially asked to represent the five objects presented in the post-interview. Children commented that they found the model useful, and some who did not use its visual aspects spontaneously, did use the language of the model to express themselves and clarify their thoughts. Nonetheless, we had the impression that there were several respects in which the model may not yet be a "natural" one for children and that it may be important to motivate the need for such a model by contrasting it with some more natural models. Further, some ideas of what models they find more "natural" come from an examination of their spontaneous drawings. In particular, in the pre-interview, the two children who attempted to represent the density of materials did so by showing materials of varying shades of gray or with varying layers of color. Such a model has the advantage that it portrays materials as well as densities as being essentially continuous--which seems closer to what children encounter in everyday life. Further, it adopts only a qualitative depiction of density, which is in keeping with the child's level of concern. And in many respects it is a very good model, one that can be used to develop their qualitative understanding of density as an intensive property. Our model, in contrast, may seem to have too much unmotivated baggage. Thus, it may be useful to begin with a model more like the ones developed by some of the children; we could then discuss the model's strengths and limitations. One obvious limitation is that the model does not allow one to represent information about size, weight, and density quantitatively. We could then present a problem which calls for quantification so that children become aware of this weakness in their model and then introduce our model as one way of portraying these dimensions more quantitatively.

Overall, we remain convinced that our approach to introducing upper elementary school children to density by involving them with modeling is a very sound, as well as a very rich one pedagogically. The children looked forward to the classes and showed some abilities to appreciate the use of computer as a modeling device. Next year we plan to build on what we have learned and develop a more extended unit that will not only give them greater time to build a concept of density but more time to appreciate at a metaconceptual level the role of models in science.

CHAPTER 5

SOME THOUGHTS ON NEXT YEAR'S TEACHING

From our experience this year, we have learned several lessons that can help us reshape our plans for the teaching experiment next year.

(1) Most students have a very good conception of material. They know how to distinguish among different materials, and they perceive each material as having distinct and specific properties such as hardness or color. Therefore, in promoting understanding of density as a local property, we will build on the notion of material kind.

(2) From a motivational point of view, the sinking and floating programs seem much more compelling to the students than the Weight/ Density program. After the post-intervention interviews, we asked students to comment on what they liked most about the teaching sessions. They agreed nearly unanimously that the sinking and floating programs and activities were the best part. Consequently, we will try to use this phenomenon as a framing context for introducing density and modeling, rather than developing these concepts separately or in isolation.

(3) We found that not all students had a clear understanding of metaconcepts. When students are given a task that requires them to implement metaconcepts such as ordering, or finding a general rule, we think it will be helpful to support the task, not only with explanation, but with a set of related activities. These should be arranged in increasing order of difficulty from very simple to more complicated and pegged to different contexts, starting close to the students' every day experiences and gradually guiding them to our subject. For instance if students have to order, we might start with 3 objects and then move to more. Or, if we ask them to look for rules, we might start with rules they use in games. These techniques will help establish terminology and make concepts explicit.

(4) So far, our lessons have avoided discussion of volume by using the "size" variable instead. However, many students do not assume that size means volume, and we now think it will be impossible to continue in this way without risking confusion about what we mean by size. This problem is most pronounced in the ordering tasks and in relation to floating boats where students need to deal directly with volume. Next year we will introduce the concept of volume in more explicit ways, perhaps by using a variation of the sinking and floating programs to do water displacement.

(5) Although most students understood density as dots per size unit in our model, some also confused weight with dots per size unit. In helping them shift attention to the concept of density, we should also reinforce weight as a separate dimension, one that they are already familiar with. We will therefore also give problems that highlight weight. For example, we might have students build or change objects on the screen, so that their weights are equal, even though they are made of different materials.

Since students will have to think about density more quantitatively in order to solve this type of problem, we could institute a more flexible range of material kinds in the program. This would make it possible to change the number of dots/size unit of available materials beyond the current range of one to five, and in different ratio if needed. Because it is difficult to find real materials with densities specifically in the 1:2:3:4:5 ratio, this modificaton would also afford greater flexibility and accuracy in representing real materials.

(6) This year's teaching also showed that students needed more work on modeling. Their "picture codes" were sometimes inconsistent and/or indistinguishable from ordinary pictures. We would like to spend more time working on the criteria for a good model and how models might differ from pictures.

(7) This year we found ourselves "presenting" the models and materials to students more than we think is desirable. In the future, we will strive to create a classroom environment in which we raise some initial questions, while leaving as much room as possible for students to do their own investigating, exploring of materials, and question raising. The issue of finding a middle ground between a lot of structure or constraints and unstructured free discovery is an important one to address. We will be as well prepared as possible to conduct or facilitate unstructured and unexpected inquiries. The few cases in which this happened this year were the liveliest and most exciting. We also learned about some of students' spontaneous models for representing density that can be incorporated into our lesson plans.

REVISED PLANS FOR NEXT YEAR (UNITS 1 & 2)

To integrate all this learning into our lesson plans, we plan to start from some phenomena of sinking and floating and raise a real-world question: What sinks and what floats?

Each child will have a kit with a tub of water so that he or she can do experimentation and collect data with all kinds of objects and materials. We will ask students to find all the "sinkers" and "floaters." We can also ask children to bring objects and materials that they find (limited to homogeneous, bulky objects) and divide them or classify them according to the "sink or float" criterion. We would want to include objects made of the same material in different sizes.

Once they have accumulated some information about the behavior of real materials, we will encourage students to look for a rule. In a way, what we are trying to teach, from a scientific point of view, is not the theory of sinking/floating, which includes an explanation of the phenomena. Rather, we are trying to find an indicator; that is, we want to find the relevant property of materials that will enable us to predict what will sink and what will float, without explaining why. We want to answer the question, "when do things sink or float?" and not, "why?" The basic process for doing this is sorting and classifying, and the process has two interdependent elements: the search for the criterion itself; and the very process of classification once we know the criterion. Both elements are on the metaconceptual level and deserve special attention.

As they search for a rule, children will express their ideas about what rules are, what they mean, and how one checks whether or not a particular rule is correct. The meaning of contradiction or conflicting evidence will come up in these discussions. We may be able to use games, sports, the legal/courtroom environment, or other metaphors to develop these ideas.

This part of the unit should be developed fully to ensure that students have a good preliminary understanding of the meaning of a rule. Since the rule we are looking for is a "sorting-rule" for nature, we will look for simple activities and examples from students' immediate environment where one applies sorting-classification rules, and then turn attention to the problem of sinking and floating.

Once students have suggested some rules for sinking and floating, the next step will be to check them experimentally. They should suggest experiments, which they will perform themselves, to validate or invalidate each suggestion. These experiments will generate more data that will be collected in the students' notebooks. The materials for the Elementary Science Study "Clay Boats" unit include a scale and other components that might be adapted to our purposes.

During this process of answering the question, what sinks and floats, we will collect students' ideas in written form, sort them, and present them to the class. This procedure -- collecting student responses and presenting them, or having students debate their positions -- will be repeated several times during our entire series of lessons. Students may draw a tentative conclusion at this point that the sinking/floating criterion has something to do with material kind.

We know that children perform best when working with a limited number of variables. In our case, there are many variables which could all be checked among different kinds of materials and within materials (of different sizes, weights, colors, and so on). To avoid confusion, we will proceed in a structured way.

To provide structure we will propose to students an additional systematic process of classification, not dividing the objects into sinkers and floaters, but reorganizing them into families according to their material kind (for example, placing all pieces of aluminum or steel in one group). At this point, we can use sink-float in a different way, to see whether it can help us distinguish materials. If two materials look the same (for example, two pieces of painted wood, or a lump of wax and a lump of clay that are the same color) can sink-float help us classify them?

After reorganizing objects into families, we will use experimentation to eliminate size, weight, and shape as factors in predicting what will sink and what will float within a family. It will not be necessary yet to define volume because when comparing two objects of different size, no matter what definition of size is used, the experiments will show that the result (sink or float) is independent of size (within the family).

Once everyone agrees that material kind alone defines an object's behavior, we will have another example of a "local property" that is discovered and demonstrated in experiments: that is, "sinkingness" or "floatingness." We will then have a working theory: materials can be divided into families of material kind where some families are sinkers and some are floaters. Once students find out from books or experiments which family a given object belongs to, they will know about all its relatives, that is, other samples of the same material. This rule works perfectly in a constant liquid. (We have not yet introduced the role of the liquid: we suspect it will be best to postpone this discussion until a later stage and to examine all the phenomena in one liquid: water.)

Next, we will want to see whether this rule can be generalized across families. First, from a classification point of view, we will now form two tribes: the sink tribe and the float tribe. Tribes are made of families, and families are made of members. We want to help students discover whether all the families that belong to one tribe share a common characteristic.

What is it about a specific material that makes it a floater or a sinker? Our intuition is that chances are very low that a student will come up with density as the parameter. Nevertheless, we will be prepared to handle questions and answers about the heaviness of a material. We will introduce the idea of crowdedness in preparation for density and spend some time clarifying crowdedness as an intensive property that is a function of the two elements, weight and size.

Assuming that density will not be a readily available criterion, we might also raise the question: What if we don't know all the information needed to make a judgment? Sometimes special tools are needed to make hidden things clear. X-rays help us see inside suitcases at the airport. Sometimes we need to use imagination to explain or describe nature and events. We might suggest that students already have a feeling for what distinguishes sinking and floating material kinds, but that they don't as yet have a name for it. Indeed, there may be properties of materials they have never discussed or thought about before.

We might start this phase by asking students to represent those properties of a tribe member that make it belong to the tribe. At this point we will introduce the idea of building visual models that depict or represent some of the properties that they come up with as characteristic of different tribes, including the concept of crowdedness that we introduced.

Again at this point we will repeat the process of asking for students' suggestions, this time about how to represent crowdedness, and we will connect this to discussions about representations in general. We will draw on other contexts where representations and crowdedness play a role. We can move gradually from icons that are used on the highways, for example, to maps, and to more abstract forms. This concept must be very carefully developed. We will spend time on the meaning of crowdedness as an intensive quantity and on modeling crowdedness in real life situations: hotels, their homes, stores, grocery store shelves, concert halls, and beaches. We want to make sure that students distinguish between total amount and amount per size unit.

We can use also multi-sensory approaches to develop the concept of crowdedness -- feeling it (arrangement of people in the room), hearing it (beads in a box, like a maraca), seeing it (intensity of color in tinted water), tasting it (degrees of sweetness or saltiness). We can talk about "packedness" and relate this to weight. We might also look at and use powders, sawdust, and/or sand to see how materials keep or lose their properties as they are ground or observed in differing amounts.

Only after these discussions and experimentation with real materials can we present the computer model as a tool to solve problems that deal with the two dimensions of total amount and amount per size unit, for example, candies and candies per bag. Now, we can use candies and bags of different sizes or other devices with which students can build physical representations for crowdedness. We may be able to do this cubically, with a cubic size unit or other three-dimensional unit standing for the two-dimensional square on the screen.

After this preparation, we may approach the specific question of the "crowdedness" of materials. This concept can be linked to the idea of "fairness" that children are already familiar with: if we are going to compare the crowdedness of two fields, we would want the fields to be the same size; then we could count the people. If we are comparing the number of chips in chocolate chip cookies, we need to use equal size cookies and then count the chips. For the time being, in order to be fair, we must compare the weights of equal size pieces. Later we will learn a fair way to compare when the pieces are not of equal size.

This discussion will introduce the idea of controlling a variable. This is an important concept, and it is crucial that students recognize and understand the variable (size, in this case) being controlled. In this year's teaching, we found the computer model to be useful in helping them understand why we hold some parameter constant. Students readily grasped the idea of comparing individual squares to figure out density.

By this point we expect to be able to introduce the idea of density of materials (homogeneous and bulky) and liquids and to look for ways to define the density of the materials students have gathered. We will be trying to help them discover whether this is an intrinsic property too. We will concentrate on finding some methods or procedure to determine density. The computer representation will help them understand why these procedures are valid.

Depending on students' grasp of the concepts up to this juncture, we may or may not be able to move on to the process of comparing the number of times a size unit can be included inside an object. In general, we wish to build from a solid qualitative understanding of density to a more quantitative one. To motivate the transition, we might pose the problem of what happens when we cannot compare equal portions of materials? Students are already familiar with the mathematical operation, division, that allows us to break the physical barriers.

Now we will return to the floating puzzle and reformulate the rule. What is it about the materials that makes them sinkers or floaters? By now, students will have an answer: the density of the materials. To complete the picture, we will now introduce the role of the liquid, first as an experimental fact, and then as a way to show how we must sometimes modify a rule to accommodate new facts. Here we can use the computer ulation to summarize and reinforce the lessons and to increase students' understanding of the quantitative aspects of the phenomena.

We can conclude the unit with some related historical stories and episodes -- for example, Archimedes's puzzle, or special materials that are extremely dense or the opposite, perhaps sinking and floating balloons. The 3-2-1 Contact television series also contains some segments (one on measurement, for example) that would provide an appropriate conclusion to this unit.

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APPENDIX A

Interview

INTERVIEW

NAME_______________ M/F DATE_______________

Ordering by weight, size and density

Materials

FIRST SET:(All pieces are 1 cubic cm) 1. AL cube/ 2. (5)AL cubes in a 'square'/ 3. (5)AL cubes in a tower / 4. (3)steel cubes/ 5. Copper cube/ 6. (7) rubber cubes. On balance scale, 1 wood = 2 rubber, 1 AL = 2 wood, 1 steel = 3 AL, 1 copper = <1 >2 steel.

2ND SET: (Cylinders are 1.5 " in diameter) 6. 2" steel cylinder (1 lb) / 7. 6" AL cylinder (1 lb) / 8. 3" AL cylinder (.5 lb).

YOU'LL BE ASKED TO ORDER THINGS IN DIFFERENT WAYS. FEEL FREE TO HANDLE THE OBJECTS IN ANY WAY YOU LIKE. HERE IS A SCALE AND A MEASURING TAPE THAT YOU MIGHT WANT TO USE TO HELP YOU DECIDE WHERE TO PUT THE OBJECTS.

A) WEIGHT

DOES THIS RUBBER CUBE HAVE WEIGHT? HOW DO YOU KNOW?

If no: DOES THIS GROUP OF 10 RUBBER CUBES HAVE WEIGHT? HOW DO YOU KNOW THAT?

I WOULD LIKE YOU TO PLACE THESE OBJECTS (first set) IN ORDER ACCORDING TO THEIR WEIGHT. PUT THE HEAVIEST ONE HERE AND THE LIGHTEST ONE THERE: IF TWO THINGS ARE THE SAME, PUT THEM ONE IN FRONT OF THE OTHER. (Note strategy)

HOW DID YOU DECIDE TO PLACE THEM IN THAT ORDER?

NOW I WOULD LIKE YOU TO ADD THESE OBJECTS (second set) TO THE GROUP ACCORDING TO THEIR WEIGHT. (Note strategy)

B) SIZE

I WOULD LIKE YOU TO ORDER THESE OBJECTS (first set) ACCORDING TO THEIR SIZE. PUT THE BIGGEST ONE HERE AND THE SMALLEST ONE HERE; IF TWO THINGS ARE THE SAME, PUT THEM ONE IN FRONT OF THE OTHER.

HOW DID YOU DECIDE WHERE TO PLACE THEM?

NOW I WOULD LIKE YOU TO ADD THESE OBJECTS (second set) TO THE GROUP ACCORDING TO THEIR SIZE

C) MATERIAL KINDS

NOW I WOULD LIKE YOU TO GROUP THESE OBJECTS BY THE KIND OF MATERIAL THEY ARE MADE OF

HOW DID YOU DECIDE WHICH OBJECTS ARE MADE OF THE SAME MATERIAL?

(Provide names & correct sorting)

D) DENSITY

(Pre-interview only)
HAVE YOU HEARD OF THE WORD DENSITY?

(Pre-interview version)
SOME OBJECTS ARE MADE OF A HEAVIER KIND OF MATERIAL THAN OTHERS. I WOULD LIKE YOU TO PLACE THESE OBJECTS (first set) ACCORDING TO THE HEAVINESS OF THE KIND OF MATERIAL THEY ARE MADE OF, THAT IS, ACCORDING TO THE DENSITY OF THE MATERIAL. PUT THE ONE WITH THE HEAVIEST (DENSEST) KIND OF MATERIAL HERE AND THE ONE WITH THE LIGHTEST (LEAST DENSE) KIND OF MATERIAL HERE; IF TWO THINGS ARE THE SAME, PUT THEM ONE IN FRONT OF THE OTHER.

(Post-interview version)
I WOULD LIKE YOU TO PLACE THESE OBJECTS (first set) ACCORDING TO THE DENSITY OF THEIR MATERIALS. PUT THE ONE MADE OF THE DENSEST MATERIAL HERE, AND THE ONE MADE OF THE LEAST DENSE MATERIAL HERE. IF TWO ARE MADE OF MATERIAL WITH THE SAME DENSITY, PUT THEM TOGETHER.

HOW DID YOU DECIDE WHERE TO PLACE THEM?

NOW I WOULD LIKE YOU TO ADD THESE OBJECTS (<u>second set</u>) TO THE GROUP

[(Pre-int. only)
ACCORDING TO THE HEAVINESS OF THE MATERIALS THEY ARE MADE OF, THAT IS]
ACCORDING TO THE DENSITY OF THEIR MATERIAL.

[Post-interview only]
DO YOU THINK THERE IS A DIFFERENCE BETWEEN WEIGHT AND DENSITY?
WHAT IS THE DIFFERENCE?

<u>Models of size, weight and density</u>

<u>Materials</u>:
Three same size cylinders (5 " high): one wax(5 oz), one AL (14 oz), one steel(2.5 lb); one steel cylinder (2 ") equal in weight to AL one.

LET'S EXPLORE SOME OF YOUR IDEAS ABOUT WHAT MAKES OBJECTS WEIGH WHAT THEY DO. THIS OBJECT (<u>large steel</u>) WEIGHS MORE THAN THIS ONE (<u>small steel</u>). HOW CAN THAT BE?

THESE OBJECTS (<u>same size wax, AL & steel</u>) ARE ALL THE SAME SIZE BUT THEY HAVE DIFFERENT WEIGHTS. HOW CAN THAT BE?

WHAT ABOUT THE DIFFERENT MATERIALS MAKES THEM HAVE DIFFERENT WEIGHTS?

THESE OBJECTS (steel & AL of equal weight) ARE DIFFERENT SIZES AND MADE OF DIFFERENT MATERIALS, BUT HAVE THE SAME WEIGHTS. HOW CAN THAT BE?

WE HAVE BEEN TALKING ABOUT SOME OF THE WAYS IN WHICH THESE 4 OBJECTS ARE DIFFERENT AND SOME OF THE WAYS IN WHICH THEY ARE THE SAME. MAKE UP A PICTURE CODE THAT SHOWS WHAT WE'VE BEEN TALKING ABOUT (SOME OF THE PROPERTIES OF THESE OBJECTS). USING YOUR PICTURE CODE, DRAW A PICTURE OF THESE 4 OBJECTS.

WHAT INFORMATION DO YOU HAVE ABOUT EACH OBJECT IN YOUR PICTURE?

HOW HAVE YOU REPRESENTED THAT INFORMATION?

[Post-interview only]
NOW I'D LIKE YOU TO DRAW ANOTHER PICTURE OF THE FIVE OBJECTS, WITH THE SAME KINDS OF INFORMATION WE'VE BEEN TALKING ABOUT, BUT THIS TIME I WANT YOUR PICTURE TO LOOK AS THOUGH YOU HAD USED THE COMPUTER PROGRAM TO DRAW IT.

DESCRIBE PICTURE: WHAT INFO?____________________________________

HOW REPRESENTED?____

DO YOU THINK THAT IS A USEFUL WAY? WHY?

[Post-int. only - DO YOU THINK THESE ARE USEFUL? WHY? DO YOU THINK ONE WAY IS MORE USEFUL THAN THE OTHER? WHY?]

Sink and float

Materials:

Tub of water / Floating objects: a) pine (1" thick): big piece, 7" x 4" & small piece, 1.5" x 4"); b) solidified glue: big piece, irregular, approx. 2.5" x 3.5" & tiny piece) / Sinking objects: c) lignum vitae (1" thick): big piece, 7" x 4" & small piece, 1.5" x 4"; d) pieces of clay: big one, circular 2" & tiny one.

YOU MAY LOOK AT THESE OBJECTS AND SEE HOW THEY BEHAVE IN THE WATER

WHAT KINDS OF THINGS FLOAT AND WHAT KINDS OF THINGS SINK?

CAN YOU COME UP WITH A RULE THAT WOULD ALLOW US TO PREDICT WHAT WILL FLOAT AND WHAT WILL SINK

Small wax (.75" diameter & 1" length) floating

WOULD THIS BIG PIECE OF WAX (2" diameter & 4.75" length) FLOAT OR SINK? WHY?

Large AL (1.5" diameter & 8.5" length) sunken

WOULD THIS PAPER CLIP SINK OR FLOAT? WHY?

Materials:

One glass with 5 oz of salt water (blue) and one glass with 5 oz of fresh water (red); a piece of lucite (.5" diameter & .5 "

CAN YOU THINK OF A REASON THIS OBJECT IS FLOATING IN THIS LIQUID AND NOT IN THAT ONE?

Post-interview only

Materials: A student drawing in which density is inversely related to the amount of air holes

COULD WE USE THIS MODEL OF MATERIALS IN THE SAME WAY THAT WE HAVE BEEN USING THE COMPUTER MODEL? WHY? HOW DO THEY COMPARE? DO YOU LIKE ONE WAY BETTER THAN THE OTHER?

BEFORE YOU GO, WE WOULD LIKE TO KNOW WHAT YOU THOUGHT OF THE LESSONS AND THE COMPUTER PROGRAMS. PLEASE BE HONEST BECAUSE WE REALLY WANT TO KNOW WHAT YOU THINK, AND WHAT YOU TELL US WILL HELP US MAKE THE LESSONS BETTER.

WHAT DID YOU LIKE THE BEST?_____________________________________

WHAT DID YOU LIKE LEAST, OR FIND BORING?________________________

ANY OTHER COMMENTS?____

APPENDIX B

Teaching Sessions

FIRST CLASS (Introduction to the Computer Program)

I. Worksheet - HOW TO USE THIS PROGRAM

II. Worksheet - WORKSHEET PROBLEMS

SECOND CLASS (Using the Program to Order and to Model)

(Have "Modeling with Dots" program loaded on all computers. Turn off monitors, students face demo computer in front of room.)

I. REVIEW PROGRAM / Discussion / 5-10 mins.

A. How do you run or use the program?

You have to look at the screen,
it tells you what to do (move arrows, press space bar, etc)
it shows you a menu from which to choose an option by typing the first letter.

If you don't see what you want to do on the menu, then use the space bar or escape key (or ask the teacher)

B. What kinds of things can we do with the program?

Build things.

Change the objects
Size, material

See them in another way
Filled in color, or seeing the grid and dots

Exchange objects, make them trade places

Get data about the objects
Size, number of dots, number of dots per block or size unit

C. It's important that you understand the three kinds of data. We will do some more problems on this in a moment.

If I talk about the "dots" in an object, is it clear what I mean? Do I mean the total number of dots, (the number of dots altogether) or the dots per building block (the dots in each size unit?) We need to be clear about the difference.

D. Any questions.

II. FREE TIME / 5 mins.

Take a few minutes to play with the program, and review the way it works. Then we will finish up the worksheets. After the worksheets, we will discuss them and then move on to another kind of activity with the computer.

III. Discussion - WHAT DOES IT MEAN TO ORDER?

A. - highest to lowest
- lowest to highest
- first to last
- biggest to smallest

B. - a "basis" for ordering
- how shall we order? according to ...

C. - what are some ways to order people in the class?
- alphabetically, by height, by age...

IV. ORDERING WORKSHEETS

A. We are handing back one of the worksheets from Wednesday (WORKSHEET PROBLEMS). If you have started the worksheet already, or even if you think you have finished it, please listen because one of the problems has changed. Double check your answers. Read along:

Instead of "What is smaller about the first one than the last one?" It is: What was the "basis" for your ordering? You ordered them according to what? What did you look at, or pay attention to when you ordered them or decided where to put them?

B. Hand out second worksheet (A FEW MORE ORDERING PROBLEMS) when first one is finished.

C. Go over the orderings from the first sheet.

1. Draw the pictures on the board from the 1st sheet.

   Discuss orderings by size, total dots, dots/size u.

   Bring up other issues:
   - some people said inches - inches measure length. Let's agree to use size units as standard measure for the total size of an object.
   - what is the total size of the objects on the board?
   - is it right to say one of them has 3 dots, or should we say "3 dots per block" or "3 dots per size unit"?
   - if we change the material of this one, what will change? (dots per block AND total dots)

2. Go over the second worksheet (have answers and data ready)

3. Questions?

V. MODELING

A. Half of you will do some problems with pennies, and the other half will do problems with beads. Then we will switch.

(Hand out materials - bags of 30 pennies OR 25 beads and 7 boxes for each group - and worksheet (MODELING WITH THE COMPUTER). You will do some problems and then think of a way to use the computer to draw a picture. Do you know what "represent" means? (For now - to show, to draw a picture.)

B. See how it goes...when all are finished (or as each group finishes?) with these tasks, explain the next part..make up a situation that the computer could represent or model. Write it down. The computer can draw certain kinds of pictures, what in the real world could these be pictures for? Discuss answers.

C. Go on to the next part when all are finished. Hold up the equal size pieces of steel and aluminum. Remember what these are? Show how much they weigh on the scale (1 pound and 1/3 lb.)

Think of a way to represent these on the computer. Put one object in one window and the other in another window. Copy the screen onto the paper and write down (show on board) ALUMINUM under one and STEEL under the other.

Next time we will have a discussion about the work you did today and do some more activities.

THIRD CLASS (Discussion of Modeling)

I. Discussion of Models - BEADS AND PENNIES

Now we want to discuss how you have used the computers to represent the beads and penny problems and what makes something a good model (or representation) of these problems.

A. Looking over your papers, we saw two main ways people represented these problems. Let's see if we all understand these models and whether someone had a different model that we should add to the chart.

1) First model
Read the problem; what information is represented in the model? How is it represented? (put on blackboard)

What	How
# of boxes	# of squares (or si· units)
# of beads	total # of dots
beads/box	# dots/size unit

note: essentially same type of code can be used for pennies:

# of piles	# of squares
# of pennies	total # of dots
pennies/pile	# dots/square

note: we have used a kind of code:

- is it used accurately? (yes)
- is it used consistently? (yes)
- does it have all the relevant information for the problem?
- does it encode color; spacing of boxes; why or why not relevant?

2) Second model
Read problem; what information is represented and how for this model?

# of beads	# of dots; # of squares
# of boxes	# of columns
beads/box	# of squares/column

- does this model have same information as the other?
- is it used accurately?
- is it used consistently?
- is one a better model of the problem than the other? Why or why not?
- do you think it is useful to make models like this? any advantages over just words? any advantages over the real thing?

Did someone else have yet another model they would like to share?

B. Mis. 1 Models (poster shows Alice represented with Model 1 and John with Model 2)

- [illegible] this be a good model?
- [illegible] s the problem? (for Alice's pile used one co[illegible]ition; for John's pile another; haven't used code cons[illegible]tently)

- other models for discussion (what's wrong)
- model where code is not used accurately
- model where all relevant information is not shown (just # of piles, nothing else)

II. MAPS (Small groups, each gets one map: Boston subway, Boston street, Boston area highway, Boston buildings souvenir)

These are all maps of Boston. Maps are something like models. Certain kinds of information are represented in certain ways. Let's now think a bit more about what makes a good map of Boston.

A. - What might each map be good for?
- Why might we use a map instead of the real thing?

B. - What are the qualities of a good map? (put on board) (easy to read or understand, accurate, consistent, has information needed for a given purpose, has quantitative info where needed)

C. - Look at your maps
- What kind of maps do you have?
- Report on WHAT info is on your map and
- HOW that info is represented
- Does yours seem like a good map?
- Is info the same on all maps? Represented the same way on different maps? Consistent within one map?

D. - Is one map a better map than the others? Why or why not?

E. - What are the limits of a map's usefulness?

F. - Summarize characteristics of good map.

FOURTH CLASS (Modeling Real Materials)

I. Construction of computer model of steel and aluminum

A. Remember when we asked you to think of a way to use the computer to model two pieces of steel and aluminum. Many of you came up with some good ideas, but it was a hard problem. Today we wll think about it in a more step by step way.

B. Show little cubes made of aluminum.
- show one single block and group of 3 blocks
- ask students to model this (let them do it - pick same material, represent different sizes)
- discuss answer and agree on the size dimension and how to represent. Put correspondance on the board.
- show large cube made of 8 small cubes
- how to represent this? shape vs. size. let them try.
- discuss - computer shows only one layer - how could we really know how many cubes there are? need to sacrifice shape. If computer were showing rooms in a hotel, would be more important to show size than shape.

C. Show one cube of steel and one cube of aluminum.
- how to represent these? agree to keep the established size dimension, and now add in next dimension, material Show on scale that steel cube is heavier than aluminum cube. (Later we will refer to density of material)
- try
- discuss - same size, different materials. Discuss rationale for different number of dots/size unit: heavier kind of materiel.

D. How much heavier is steel piece than aluminum piece exactly? How many equal size pieces of aluminum would weigh the same as an equal piece of steel?
- show on scale that 3 al. cubes balance with 1 steel
- how can we represent that idea on the computer?
- agree to let each dot stand for unit of weight
- find the materials on the computer which would allow us to have the same number of dots in each object, but one object is 3 size units and the other is 1 size unit

E. Discussion
- Go over the three dimensions
- size, weight, density - put on board with corresponding ways of representing on computer
- Discuss difference between weight and density
- Discuss objects that weigh the same, but have different densities
- Discuss crowdedness - (people in hotels, chips in chocolate chip cookies)

F. - Review how we modeled steel and aluminum cubes
- now we will model some bigger pieces of steel and al.
- show again the equal size cylinders of steel and aluminum and tell weights: steel=1 lb, al=1/3 lb.
- how many equal size pieces of aluminum are necessary to equal the weight of the one piece of steel? (3)
- try to model these cylinders on the computer (note they are the same size, different weights)
- collect copied drawings

FIFTH CLASS (Review and Discussion of Usefulness of the Model)

A. Review (5 mins)
- draw 3 al. cubes and 1 steel cube on the board
- remember that they balance
- review and write on board: size blocks - size
  total dots - weight
  dots/block - density (weight in a size unit, how much weight is packed into a size unit)
- put drawing of 3 al and 3 steel on board for comparison
- how does code inform us about density?
- which object would weigh more?
- how many groups of 3 al would equal the group of 3 st in weight?
- showing relatively same size or twice, three times the size or weight, not exactly how big or exactly the weight
- just need to be consistent
- might incorporate transition from small ind. cubes to big solid piece

B. Just constructed models. Explore. (10 mins)
- here are some of the models you have drawn (poster on board) which do you think is the best model. why?
- if steel is represented with 3 d/c when modeling cubes, why do you think it should be 5 d/c when modeling large pieces?
- have we shown that the objects are the same size?
- have we shown the different weights?
- if 1 pound is represented with 30 dots, how m[illegible] dots would represent 1/3 pound?
- say which model we prefer and why (shows equal size and numeric weight relationship i.e. 1:1/3)
- discussion of unit
- do you think this way of representing objects is useful? how?

C. Tell our purpose: (5 mins)
we know students have difficulty distinguishing weight and density and understanding how they are all inter-related.

Can anyone say something about the different between weight and density? Can you say something about the difference without relying on the dots analogy? We can think of density as how much weight (matter) is packed into a given size space.

Our purpose is to develop a model which clearly portrays different kinds of quantities and shows how they are inter-related.

D. Look at units of size and weight: they are not the same
- 1 dot; 1 square
  - you can have things that are the same size, but different weights (draw on board)
  - model shows visually how that could be.

- can think of this as more weight (matter) packed into a given space
- draw three pictures on the board, just by looking we can tell which is heaviest, biggest, made of densest material

One thing our model might use is a key to knowing how much weight a dot stands for: one pound? one ounce?)

- notice
  - both the size and weight of an object increase as you add more to it
  - adding more squares necessitates increase in both size and weight

- what stays the SAME?
  - density. is that surprising?
  - if I keep adding building blocks, does the total weight change (yes, increases)
  - density is characteristic of the material rather than of the whole object
  - is the density the same here as there?
  - how do we know density is constant throughout the material?
  - density is constant, and is the same throughout the whole object.
  - the density is the same on the top and on the bottom
  - the density of the material is the same whether we take a little piece or a big piece
    - show three pieces of aluminum (small, med. large)
    - them around the room
    - is the density of the material the same in all three pieces?
  - how could you convince yourselves (prove to selves) the material in a little piece of aluminum has the same density as that in a large piece - compare to same # of dots/size unit.
  - (break them up? what if we took the same size pieces?)
  - demonstration with clay
  - one little piece of clay has a certain density (compare with small piece of steel - which weighs more? steel. which is denser? steel)
    - does large piece have same density?
    - what has changed? the weight and the size. compare to computer program.
    - show very large piece of clay. it is heavier than the little piece of steel, but it is not as dense as the steel

E. Let's also compare our models to some of yours developed in the pre-interview to see if it serves this specific purpose better than your own models
- we asked you in the individual sessions to you in the explain why objects weigh what they do. Initial questioning revealed that most of you thought
  - the size of an object
  - the kind of material an object is made of
    both affected its weight
  - some said color, hardness

And then to draw a picture that would show some of your ideas about the ways in which certain objects were the same and different with respect to their material, size and weight.

F. First let's discuss what it was about size and material that could affect the weight of objects

- two objects made of same material, but different sizes more of the material, it would weigh more
- two objects the same size made of different materials
- some ideas
  - some material is just a heavier kind of stuff (density) - some materials are darker and therefore heavier (vulcanite test: compare weights of same size rods made of dark color rubber with light color aluminum; aluminum is heavier and denser)
  - some materials are harder and therefore heavier (lead strip compared to steel strip. Lead is denser and heavier, yet is soft and pliable)
  - some objects were empty and some full (convince solid all the way through)
  - because some objects were made of heavier kind of materials, you could have a little piece of heavy material that was equal in weight to a large piece of light material (the steel and al in pre-int)

G. Let's look at how size is represented

- in our model- squares are a UNIT measure of size
- in your models - height of object
- do you think that these are equivalent - or that one has certain advantages
- consider objects of different shapes
- suppose we wanted to know how much bigger one was than another: units make comparisons easier.

H. Many of you wanted to represent material in some way. (Poster on board)

- go through different reps of material- color, intensity of shading, use
- how do these reps tell us something about weight of objects?
- about the distribution of weight
- look at weight poster
- is there an advantage to one of the models?
- uses?
- our purpose (quantification, relationships, learning tool)

I. Conclude with discussing relationship of size and material to weight

- if we know the size and material, can we predict weight (which models allow us to do that)
- (ex. material has 3 d/su and is 5 su. what is weight?)
- d/c times size = weight
- if we knew that this (on board) weighed 40 units and it had a size of 10, what would its density be?
- weight/size = density
- more examples on the board, then ask verbally (if an object weighed 100 pounds and was 50 size units, what would its density be? (2lb/size unit))

SIXTH CLASS (Individual Sessions - 2 students)

Name(s) ________________________________ Date __________

I. A. Construct the following: (have data showing with weight units and wt/sz units)

(A): height=4	(B): height=4
width=1	width=1
d/su=1	d/su=2

Give: balance scale, 5 wood rods, 3 each of vulcanite, brass, al.

Ask: I've represented 2 of these rods on the screen. Can you figure out which 2 I've represented?

____

____

____

____

(hints: Can you tell me something about their weights? How much more does B weigh than A? How many A's would be equal in weight to one B?)

B. Ask: How could you represent an aluminum rod on the screen in the third window? Can you use the wood rods to help you figure it out?

____

____

____

____

(hints: How much heavier is the aluminum rod than a wood rod? How many wood pieces this size are equal in weight to one aluminum piece this size?)

C. Give: piece of paper and pencil

Ask: If we weren't limited to five weight units per size unit on the computer, how would you represent this piece of brass? Can you use the aluminum pieces to help you figure it out?

____

____

____

(hints: Is it the same size as the other rods? Is it the same weight as the other rods? How many aluminum rods are equal in weight to one brass rod? How is the weight distributed?)

II. A. Look at objects on the screen that have density ratio 1:2

Ask: How could you make these two objects weigh the same amount by changing their size?

What changed? ____
What stayed the same? ____

Compare with real objects and guide to seeing that task could be accomplished by doubling amount of one OR halving amount of other.

B. Does the density change if we cut the object in half?

What if we cut it in half again?

Is the density of the material the same for a piece cut off the top, cut off the bottom, or a piece cut from the middle?

Can you imagine the smallest little bit of aluminum and the smallest little bit of brass?

Are they the same size in your mind? ______________________

Does one weigh more than the other?

Is one made of denser material than the other?

Is the density of the material used for that tiny bit of brass the same density of the material used for this brass rod, and this brass weight?

III. A. Density is a property of material that stays the same, no matter how much of the material we have. It is how much weight is packed into a certain size space. We can say that density is a measure of the intensity of weight. Let's compare that idea with some other examples of intensity.

B. Let's think about intensity of color. Here are three containers of water. I'll put one drop of coloring in one, three drops in the next, and 6 drops in the next.

Which has the greatest intensity of color?

Does it make a difference if I look at the water on top or on the bottom or in the middle...Does the intensity of the color change?

What if I poured half of this colored water out, would the intensity of the color change?

What if I took just a little drop of the water...would the intensity of the color change?

The intensity of the color, just like the density of a material is the same throughout, and is the same in a small sample as in a big sample.

If we make the comparison of intensity of color to intensity of weight, that is density, which cup would correspond to the densest material?

If I take two cups of water the same color and add them together, will the color change?

(Try it) Compare this to material: Even with more material, the density doesn't change.

Can you think of any other examples of intensity or packing?

If gum is 35 cents a pack and you buy one pack, and you (other) buy ten packs, who has spent more per pack? Who has spent more money in all?

Sweetness: different amounts of sugar in equal amounts of water. Each cup separately, tastes the same all the way through.

SEVENTH CLASS (Sinking and Floating, part 1)

I. In a little while we will use the computer to learn about sinking and floating. But first:

We will order some objects according to the density of the materials they are made of. (BRASS, STEEL, ALUMINUM, WOOD: have cubes of each) Show the process:

First review:
Have balance scale and show small brass cube and small aluminum rod. Place on scale and ask:
- can you tell which is heavier?
- can you tell which is made of denser material?

Then show equal size pieces of brass and aluminum and put them on the scale.
- which is heavier?
- which is made of denser material?

Summary:
To find which is denser: take equal size pieces and weigh them. If they are the same size, then the heavier one is made of denser material. Once you know that one kind of material is denser than another, it doesn't matter how much of it you have, it will always be denser than the other.

II. Establish order by weighing equal size pieces.
Write on board:
(densest) BRASS STEEL ALUMINUM WOOD (least dense)

III. How about liquids?
- Show two identical containers with same amounts of oil and water
- Where would these go in the order? How can we be sure?
- Weigh equal amounts
- Oil feels thicker, yet its not denser.
- How to compare liquids and solids
- Have equal size piece of wood, weigh it along with container, compare to weight of liquid in container.
- Write on board where oil and water go in the order
- Pass around the mystery container (sealed and wrapped container of mercury - warn students to be careful)
- What is surprising? That it is small and heavy for its size?
- It weighs one pound. Compare to one pound of steel.
- It weighs the same, yet is smaller, therefore it is denser than steel.
- Do you know what the material is? It is mercury. Mercury is a liquid. It is used in thermometers. Read about it for next time. Some people say solids are always denser than liquids. Is that true? No.

Show order on board: MERCURY BRASS STEEL ALUMINUM WATER OIL WOOD

IV. As you remember, we asked you some time ago about different objects: which sink and which float?

Before we continue experiments with real liquids and solid objects - we want you to try out a program that lets you model the liquids and the objects and perform experiments on the screen.

We will briefly show you how the program works up until the "experiment". (Show how program works on demo computer in front of room)

Go and use the program now to see if you can come up with a rule: If we assume that the program represents the real world correctly, what is the rule that will let us predict when a given object will float and when will it sink? Each of you will write your answer and then we will do some real experiments.

Hand out worksheets which say:

AN OBJECT WILL SINK IN A LIQUID IF:

AN OBJECT WILL FLOAT IN A LIQUID IF:

V. Collect the answers and discuss briefly.
Questions:
What happens when oil and water are mixed together?
Do all people float?

EIGHTH CLASS (Sinking and Floating, part 2)

At the end of the last class, we did some experiments with the computer and tried to come up with a general rule that would tell us when an object will sink and when it will float.

What do scientists do? Make experiments, figure things out.... We all notice things around us. Scientists notice things too and then try to figure out a rule which will explain why certain things happen or they try to figure out a general rule to use in order to predict what will happen in certain situations. They make experiments to test their rules or to help them discover a rule.

Examples: You are a scientist and you notice that a day in February is shorter than a day in April. (Draw a little diagram on the board) A day in April is shorter than a day in June. A day in June is longer than a day in October and a day in October is longer than a day in December. You make a general rule about the length of all the days in a year. Does someone know the rule? (Everyday is longer than the one before until June 21, then they start getting shorter again until December 21.) How can we test the rule? Does the rule let us make predictions that come true? (See if it is correct for the next year)

If you notice that 5 years from now, a day in February is very long, then you would have to change your rule. Scientific rules can change, but many of them last for a long time, often hundreds or thousands of years. People used to think that the earth went around the sun. We now know that the sun goes around the earth. The rule that we use now is about 500 years old. (Copernicus)

We noticed that some things sink and some things float and we want to find a general rule that will allow us to predict whether an object we have never seen (or tried, tested) before will sink or float.

There were a lot of ideas.

Discuss rules:

1) It depends on the COLOR
   - show rubber cube and vulcanite rod in water as counterexample, also hard glue and chalk
2) HEAVY things sink, LIGHT things float
   - show big piece of wood and small piece of clay in water as counterexample

Then some of you thought it might have to do with the weight OR the density of the object. Still others thought it had something to do with the weight or density of an object COMPARED to the liquid.

The rule that the computer uses is the same rule that applies to real objects. We will spend a little more time looking for the rule. To make it clearer, we will divide the experiments. First we will only be concerned with SINKING. When will an object sink?

Hand out worksheets - FIND SOME SINKING OBJECTS

Write down the kind of object, the kind of liquid. You will need to get some data.

Discussion while other program (Sink the Raft) is being loaded.

What are some examples?

Include 3/2 (weights are 72/96) and 5/4 (weights are 120/192)

Include 2/1 (weights 42/48) The object weighs a little.

Do you think it has more to do with the weight or the density?

Do you think it has more to do with the density of the object or the density compared to the liquid?

The next program will let you do a few more experiments...you will be able to change the size and thereby the weight of the objects to see if that will effect whether it sinks or floats.

We have another program for you to try, that will let you change the size of the objects you drop into liquids. Up until now all the objects were the same size. Use the program to see if changing the size will make a difference in whether the object sinks or floats. See if by adding more and more material and thereby increasing the weight of an object will have an effect. Or taking material away, making the object lighter and lighter will have an effect. Write some examples on the worksheet.

Hand out worksheets - CREATE A GREEN OBJECT IN WHITE LIQUID

Discuss.

Come to conclusion that in order for an object to float, the density of its material should be less than the density of the liquid.

Questions for fun:

Archimedes' puzzle - Show that a large piece of clay sinks. Then show two smaller pieces of clay. One sinks and one floats. Is one a fake? Why? (One has a piece of cork hidden inside)

Balloons - Why do you think that balloons filled with helium float? (Helium is less dense than regular air)

APPENDIX C

Worksheets

NAME(S) ______________________________ DATE __________

How to use this program

In order to do some thing on the screen there are only two things you need to tell the computer:

1) WHERE TO DO THINGS.
2) WHAT TO DO.

WHERE?

You choose where to work by moving the window frame. There are THREE work areas.

***USE THE ARROW KEYS TO MOVE THE FRAME.

***PRESS THE SPACE BAR WHEN YOU DECIDE ON A SPOT.

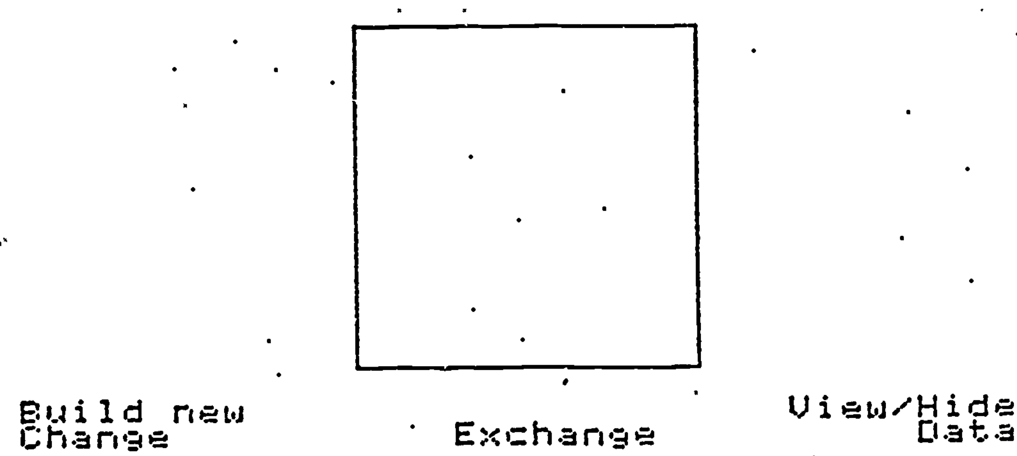
Which window did you choose? (1st/Left, 2nd/Middle, 3rd/Right)______

____

You can change your mind and work somewhere else.

***PRESS THE ESCAPE KEY.

Move the frame to the middle window and press the space bar. Your screen should look like this:



Does it?______________________

NAME(S) ____________________

WHAT TO DO?

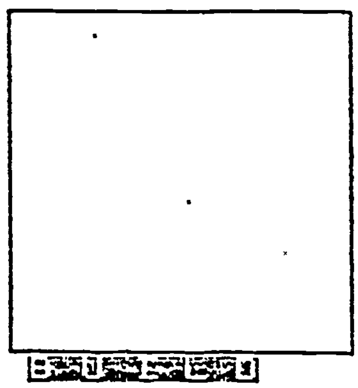
You are looking at a menu. There will be a few menus in this program. You pick an option from a menu by typing in the first letter of the word.

BUILD

Now you will build an object.

*** TYPE IN THE LETTER "B" FOR "BUILD."

The screen should look like this:



choose material

Green Purple White Orange Blue

Does it? ____________________

In order to build an object, we need to choose a "material". You see a menu with building blocks of different materials.

***PRESS "P" FOR "PURPLE."

The first building block should appear in the window.

How many dots are in the building block? ____________________

***NOW USE THE ARROW KEYS TO CHANGE THE OBJECT'S SIZE.

***USE ALL THE ARROW KEYS TO MAKE THE OBJECT BIGGER AND SMALLER.

***MAKE THE BIGGEST POSSIBLE OBJECT.

Describe the object you just built. (How tall, how wide, how many blocks?) ____________________

NAME(S) ______________

***PRESS THE SPACE BAR.

This saves the object on the screen. CONGRATULATIONS!! You have just completed the first object on the screen.

***MOVE THE FRAME TO THE THIRD WINDOW.

***BUILD THE SMALLEST ORANGE OBJECT. SAVE IT.

Describe it. (How many dots does it have? How big is it?)

____

***BUILD AN OBJECT IN THE FIRST WINDOW THAT HAS A TOTAL OF 12 BLOCKS. THEN COPY (DRAW) THE OBJECT ON THIS PAPER.

If some thing went wrong try these:

A) Press space bar
B) Press ESC
C) Call the Teacher
D) Give the computer a hug.

4.

NAME(S) ____________

CHANGE

You can change objects that are on the screen. You can change their SIZE or change the MATERIAL that they are made of. You can also ERASE them completely.

*** MOVE THE FRAME ONTO THE LARGEST OBJECT ON THE SCREEN.

What kind of building blocks does it have? ____________

Now we'll change the material to "Blue." "Blue" building blocks have five dots in them.

*** PRESS THE SPACE BAR TO SEE THE MENU.

*** TYPE "C" FOR "CHANGE".

You should now see this menu:

```
Material          Size          Erase object
```

*** TYPE "M" FOR "MATERIAL."

*** TYPE "B" FOR "BLUE."

What happened? ____________

*** FIND THE SMALL ORANGE OBJECT ON YOUR SCREEN. (orange material has 4 dots in each building block.)

*** NOW CHANGE THE ORANGE OBJECT'S MATERIAL TO GREEN.

How many dots are in a green building block? ____________

NAME(S) ______________________

Now we'll change the size of this object.

*** TYPE "C" FOR "CHANGE."

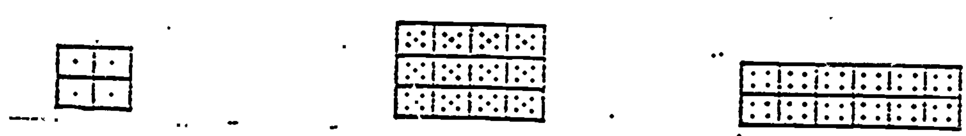
*** TYPE "S" FOR "SIZE."

*** USE THE ARROW KEYS TO MAKE THE OBJECT 4 BLOCKS TALL AND 2 BLOCKS WIDE.

*** PRESS THE SPACE BAR TO SAVE IT.

Draw a copy of this object here on the paper.

Change the objects on your screen, one by one, until the screen looks like this:



Make sure your objects look exactly like the one's above before you go on.

Now erase the middle object.

*** MOVE THE FRAME TO THE MIDDLE OBJECT AND PRESS THE SPACE BAR.

*** TYPE "C" FOR "CHANGE."

*** TYPE "E" FOR "ERASE."

NAME(S) ______________________

EXCHANGE

You can make two objects trade places on the screen by using the "Exchange" command. Exchange the objects in the first and third windows:

***MOVE THE FRAME TO THE FIRST WINDOW.

***PRESS THE SPACE BAR.

***TYPE "E" FOR "EXCHANGE."

***MOVE THE FRAME TO THE THIRD WINDOW.

***PRESS THE SPACE BAR.

VIEW/HIDE

All objects can be seen in two ways. One way is to see the building blocks. The other way is to see the object in a solid color.

***MOVE THE FRAME TO THE OBJECT IN THE FIRST WINDOW AND PRESS THE SPACE BAR.

***TYPE "V" OR "H" FOR "VIEW/HIDE."

What happened? ______________________

Use the "View/Hide" option on the object in the third window.

What happened? ______________________

Change an object's material while it is in color.

What was the old material? ______________________

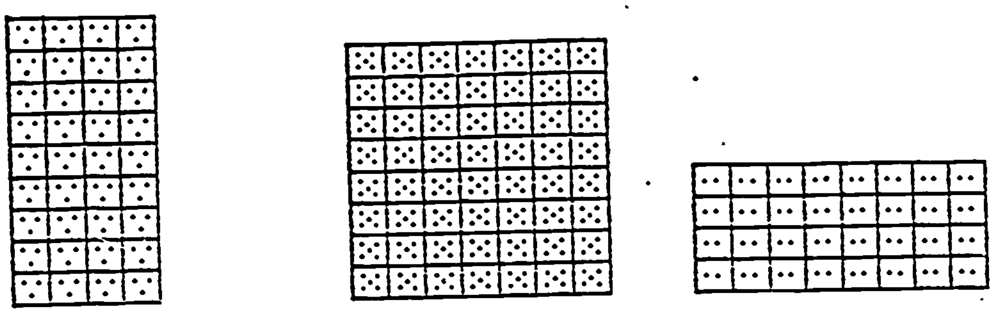
What did you change it to? ______________________

NAME(S) ____________________

DATA

The computer will do some counting for you.

First make the following objects on your screen:



Look at the object in the third window.

How many building blocks does it have? ____________________

How many dots does it have altogether? ____________________

See if you got the same numbers that the computer got:

***MOVE THE FRAME TO THE THIRD WINDOW.

***PRESS THE SPACE BAR.

***TYPE "D" FOR "DATA"

You should now see the "Data" menu.

```
Total dots     Dots per size unit     Size
space-bar                   to continue
```

***TYPE "S" TO SEE THE "SIZE" OF YOUR OBJECT.

The size is the number of building blocks. Each block is one size unit.

The object in the third window has ____________ **size units.**
Is that what you counted? ____________

***TYPE "T" TO SEE THE "TOTAL NUMBER OF DOTS" IN YOUR OBJECT.

The object in the third window has ____________ **dots total.**
Is that what you counted? ____________

NAME(S)____________________

***TYPE "D" TO SEE HOW MANY DOTS ARE IN EACH BUILDING BLOCK.

This is the number of dots per block, or the number of dots per size unit.

Get all the data for the other objects on the screen.
Which object has the most dots?____________

Watch what happens to the data as you change an object's material.

Write down the data for the object in the middle window on this paper.

Change the material of the object in the middle window.

Write down the new data.

What changed? ____________________

What stayed the same?____________________

Now change this object's size.

Write down the data.

What changed this time?____________________

What stayed the same?____________________

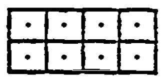
CONGRATULATIONS!!

YOU ARE NOW A CERTIFIED PROGRAM-USER.

NAME(S)__________________________________ DATE ____________

WORKSHEET PROBLEMS

CREATE THE FOLLOWING ON THE SCREEN.



GET ALL THE DATA.

Use the "Exchange" command to put these objects in an order from lowest to highest in some way.

SHOW HOW YOU HAVE ORDERED THE OBJECTS (COPY THEM FROM THE SCREEN ONTO THIS PAPER)

What is smaller about the first one than the last one?

DID YOU PUT THEM IN ORDER ACCORDING TO THEIR SIZE?

____

If not, use the "Exchange" command to order them by their size.

COPY THE ORDERED OBJECTS FROM THE SCREEN ONTO THIS PAPER.

NAME(S)

C-10

HAVE YOU ORDERED THE OBJECTS BY THEIR TOTAL NUMBER OF DOTS YET? ____________

If not, do that now, and copy the new ordering on the paper.

FINALLY, HAVE YOU ORDERED THE OBJECTS ACCORDING TO HOW "CROWDED" THE BUILDING BLOCKS ARE? (THAT IS ACCORDING TO THE NUMBER OF DOTS PER SIZE UNIT) ____________

If not, do that now and copy the objects in order on the paper.

AND NOW, FOR SOMETHING COMPLETELY DIFFERENT:

HERE ARE THREE DATA WINDOWS:

72 DOTS 24 SIZE u	3 DOTS/SZu	30 DOTS 30 SIZE u	1 DOTS/SZu	125 DOTS 25 SIZE u	5 DOTS/SZu

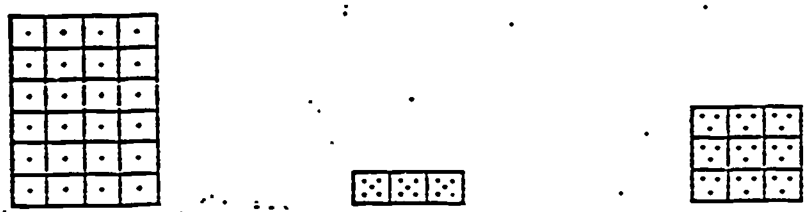
CREATE THE OBJECTS THAT HAVE THIS DATA.

COPY THEM ON THE BACK OF THIS PAPER.

NAME ______________________________ DATE ______________

A FEW MORE ORDERING PROBLEMS
(lowest to highest)

CREATE THE FOLLOWING ON THE SCREEN:



USE THE "EXCHANGE" COMMAND TO ORDER THESE BY THEIR SIZE.

(hint: Let the computer do some counting for you. Use the data option.)

COPY THE ORDERED OBJECTS FROM THE SCREEN ONTO THIS PAPER.

NOW ORDER THE OBJECTS BY THEIR TOTAL NUMBER OF DOTS.

COPY THE ORDERED OBJECTS HERE:

NAME ______________________________

FINALLY, ORDER THE OBJECTS ACCORDING TO HOW "CROWDED" THE BUILDING BLOCKS ARE, that is, according to the number of dots per size unit.

COPY YOUR ORDERED OBJECTS HERE:

NAME(S) ______________________ DATE __________

MODELING WITH THE COMPUTER

A. Alice has 4 piles of pennies with three pennies per pile. How many pennies does Alice have altogether? __________

John has 6 piles of pennies with 2 pennies per pile. Arrange the piles of pennies on your desk.

Does one of them have more pennies altogether? __________

Now use the computer.
In one window, represent Alice's piles of pennies.
In another window, represent John's piles of pennies.

Copy the computer screen on this paper.

B. Now Alice has 10 beads which she wants to arrange in two boxes with the same number of beads in each box. How many beads would she have in each box? __________

John has 15 beads which he wants to arrange in 5 groups of equal size. Put the beads in the boxes the way John wants them.

Does one person have more beads in a box than another? __________

Now use the computer to model this problem.
In one window, show how Alice's beads are arranged.
In another window, show how John's beads are arranged.

Copy your screen on the paper.

ME: ______________________ C-14

Make up another real life problem which can be modeled on the computer.

NAME________________________

FIND SOME SINKING OBJECTS.

1. Material of OBJECT:____________________
   Material of LIQUID: ____________________

IS THE OBJECT HEAVIER (MORE TOTAL WEIGHT) THAN THE LIQUID? ______

IS THE OBJECT DENSER (MORE WEIGHT PER SIZE UNIT) THAN THE LIQUID?________

2. Material of OBJECT:____________________
   Material of LIQUID: ____________________

IS THE OBJECT HEAVIER (MORE TOTAL WEIGHT) THAN THE LIQUID? ______

IS THE OBJECT DENSER (MORE WEIGHT PER SIZE UNIT) THAN THE LIQUID?______

3. Material of OBJECT:____________________
   Material of LIQUID: ____________________

IS THE OBJECT HEAVIER (MORE TOTAL WEIGHT) THAN THE LIQUID?______

IS THE OBJECT DENSER (MORE WEIGHT PER SIZE UNIT) THAN THE LIQUID?_____

****

IS IT POSS' 'LE TO FIND AN OBJECT WHICH WEIGHS LESS THAN THE LIQUID, BUT STILL SINKS? (IF SO, WHAT MATERIALS ARE THEY MADE FROM?)

IS IT POSSIBLE TO FIND AN OBJECT THAT IS LESS DENSE THAN THE LIQUID, BUT STILL SINKS? (IF SO, WHAT MATERIALS ARE THEY MADE FROM?)

NAME ________________________________ DATE __________

***CREATE A GREEN OBJECT IN WHITE LIQUID.

DOES IT SINK OR FLOAT?

***MAKE THE OBJECT AS BIG AS YOU CAN.

DOES IT SINK OR FLOAT?

***MAKE THE OBJECT AS SMALL AS YOU CAN.

DOES IT SINK OR FLOAT?

DOES CHANGING THE *SIZE* AFFECT WHETHER IT SINKS OR FLOATS?

WHEN YOU MADE THE OBJECT BIGGER OR SMALLER, DID ITS *WEIGHT* CHANGE?

WHEN YOU MAKE AN OBJECT BIGGER OR SMALLER DOES THE *DENSITY* OF ITS MATERIAL CHANGE?

***WHEN YOU ADD OR REMOVE MATERIAL YOU CHANGE THE SIZE AND THE WEIGHT OF THE OBJECT, BUT NOT THE DENSITY OF THE MATERIAL.
DID CHANGING THE *SIZE AND WEIGHT* HAVE AN EFFECT ON WHETHER IT SANK OR FLOATED?

***FIGURE OUT A WAY TO MAKE THE SMALL OBJECT SINK.

HOW DID YOU DO IT?

***THINK OF ANOTHER WAY TO MAKE THE OBJECT FLOAT AGAIN.

HOW DID YOU DO IT?

DO ALL OBJECTS FLOAT IN WHITE LIQUID?
FIND SOME THAT DO.

CAN YOU FIND SOME THAT DON'T?

DO ALL OBJECTS FLOAT IN PURPLE LIQUID?

WHICH DO AND WHICH DON'T?

***FIND AN OBJECT WHICH FLOATS IN SOME LIQUID AND SINKS IN OTHER LIQUID.

WHY DO YOU THINK THAT HAPPENS?

CAN YOU THINK OF A RULE THAT TELLS US WHEN SOMETHING WILL SINK AND WHEN IT WILL FLOAT?